Homework #1

Math 461, Fall 2021

Due Wednesday, September 1

Exercises:

1. Use the methods from our first day of class to determine which of the following figures are homeomorphic to which others. Justify your answer, though at this point we don't have the precise tools to provide rigorous proofs. (Note that you are being asked to make $\binom{8}{2} = 28$ comparisons! You don't need to compare each pair individually; part of your work is figuring out how to present the answer efficiently.)

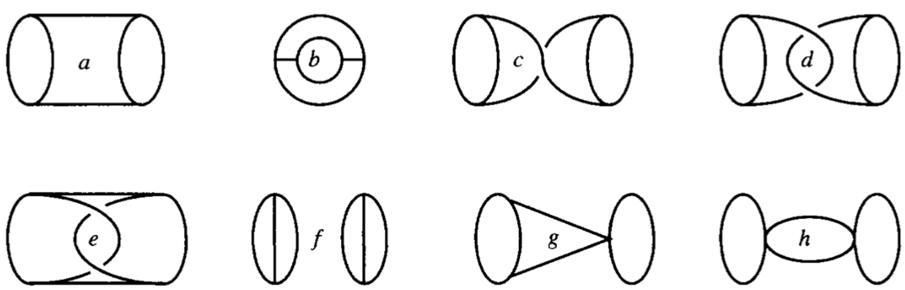

Figure 1: (From Stahl–Stenson)

2. Let $f : A \to B$ be any function, and suppose U, V are subsets of B. Prove:

   (a) If $U \subset V$ then $f^{-1}(U) \subset f^{-1}(V)$

   (b) $f^{-1}(U \cup V) = f^{-1}(U) \cup f^{-1}(V)$

   (c) $f^{-1}(U \cap V) = f^{-1}(U) \cap f^{-1}(V)$

3. Suppose $f : A \to B$ is a function. Define a relation $\sim$ on A, called the *equivalence kernel* of f, by the rule

$$x \sim y \text{ if and only if } f(x) = f(y).$$

   Prove that $\sim$ is an equivalence relation. When $f : \mathbb{R} \to \mathbb{R}$ is defined by $f(t) = \cos(2\pi t)$, what is the equivalence class of 0 under the equivalence kernel relation?

4. Define a function $d : \mathbb{R}^2 \times \mathbb{R}^2 \to \mathbb{R}$ by

$$d(\mathbf{x}, \mathbf{y}) = |x_1 - y_1| + |x_2 - y_2| \, .$$

Show that d is a metric on $\mathbb{R}^2$. Draw a picture of the open ball $B(\mathbf{0}, \frac{3}{2})$ in $\mathbb{R}^2$.