

Homework 1, due on Friday, August 28

Read carefully chapters 1,2,6 in the textbook. **It is crucial that you read and attempt the exercises at the end of each chapter as they form integral part of the book.** Solve the following problems.

Problem 1. Let $f : A \longrightarrow B$ and $g : B \longrightarrow C$ be functions.

- a) Prove that if both f and g are surjective then gf is surjective.
- b) Prove that if gf is injective then f is injective. Show by example that g need not be injective.

Problem 2. Let $f : \mathbb{Z} \longrightarrow \mathbb{Z}$ be a function defined as follows: $f(m)$ is the last digit in the decimal expansion of m . What is the domain, codomain, range of f ? Find $f(S)$, where S is the set of all even numbers.

- b) What is $f \circ f$?

Recall that if U, W are sets then

$U \cup W$ denotes the union of the sets U, W ,

$U \cap W$ denotes the intersection of the sets U, W ,

$U \setminus W$ is the set obtained by removing from U all the elements that are in W ,

$U \div W = (U \setminus W) \cup (W \setminus U) = (U \cup W) \setminus (U \cap W)$ is the **symmetric difference** of the sets U and W .

Problem 3. Let $f : A \longrightarrow B$ be a function.

- a) Prove that if S, T are subsets of A then $f(S \cup T) = f(S) \cup f(T)$. Show that the analogous statement for intersection is not always true.
- b) Prove that if S, T are subsets of B then $f^{-1}(S \cap T) = f^{-1}(S) \cap f^{-1}(T)$, $f^{-1}(S \cup T) = f^{-1}(S) \cup f^{-1}(T)$, and $f^{-1}(S \setminus T) = f^{-1}(S) \setminus f^{-1}(T)$.
- c) State and prove a result relating $f^{-1}(S \div T)$ and $f^{-1}(S) \div f^{-1}(T)$.
- d) Prove that if S, T are subsets of A then $f(S) \setminus f(T) \subseteq f(S \setminus T)$. Show by example that the equality does not always hold.

Problem 4. For a set X we define $P(X)$ as the set of all subsets of X (the book denotes it by P_X). $P(X)$ is called the **power set** of X . The operations $\cup, \cap, \setminus, \div$ are binary operations on $P(X)$. Which of them are associative? Which of them are commutative? Which have the identity element?