Math 108: Exam 1 **Name** KEY

Fall 2013 **Instructor** ______________________

Page	1	2	3	4	5	6	Total	Course Points
Points	12	16	14	24	26	21	113	150
Score								

- Calculators are not permitted for this test.
- Show your work unless the problem requires only a short answer.
- There are problems on both front and back of the pages.
- If you need scrap paper, ask your instructor. You may not use your own. If you do use scrap paper, make sure to hand it in at the end of the exam.

1. (18 points – 3 points each) For the following six questions, choose the ONE BEST answer:

When written in its equivalent fractional form, $.\overline{27} =$

(a) $\frac{27}{100}$ (b) $\frac{1}{27}$ (c) $\frac{3}{11}$ (d) $\frac{26}{99}$ (e) none of these

$(3x-5y)^2$

$(3x-5y)(3x-5y)$
$= 9x^2 - 15xy - 15xy + 25y^2$
$= 9x^2 - 30xy + 25y^2$

(a) $9x^2 - 25y^2$
(b) $9x^2 + 25y^2$
(c) $3x + 25y^2$
(d) $9x^2 - 30xy + 25y^2$
(e) none of these

Which is true about the following set of ordered pairs?
$\{(1,2), (-3,4), (7,0), (2,4), (5,-1)\}$

(a) The set represents a function having an inverse function.
(b) The set represents a function having no inverse function.
(c) The set does not represent a function.
(d) It cannot be determined from only five points whether the set represents a function.
(e) none of these

$f(x) = \frac{x^3}{\sqrt{1-x^2}}$ is an example of a function that is:

(a) odd
(b) even
(c) defined for all reals
(d) symmetric with respect to the y-axis
(e) none of these

$f(-x) = \frac{(-x)^3}{\sqrt{1-(-x)^2}}$
$= \frac{-x^3}{\sqrt{1-x^2}}$
$= -f(x)$

3. (14 points) Combine and/or simplify each of the following expressions as much as possible. Specify any necessary domain restrictions

3 a) $\sqrt{18x^2y^3} = 3xy\sqrt{2y}$

(+1) $3\sqrt{2}$
(+1) x
(+1) $y\sqrt{y}$

3 b) $\sqrt[3]{250} - \sqrt[3]{54} = \sqrt[3]{125 \cdot 2} - \sqrt[3]{27 \cdot 2}$

$= 5\sqrt[3]{2} - 3\sqrt[3]{2}$

$= 2\sqrt[3]{2}$

(+1) $5\sqrt[3]{2}$
(+1) $3\sqrt[3]{2}$
(+1) answer

4 c) $\frac{x^2+x-6}{x^2+7x+12} \cdot \frac{x^2+3x-4}{x^2-4} = \frac{(x+3)(x-2)(x+4)(x-1)}{(x+4)(x+3)(x+2)(x-2)}$

$= \frac{x-1}{x+2}$ if $x \neq -4, -3, 2$

(+2) factoring (+1) cancellation (+1) Domain restriction

4 d) $\frac{3x}{2x-3} + \frac{3x+6}{2x^2+x-6} = \frac{3x}{2x-3} + \frac{3x+6}{(2x-3)(x+2)}$

$= \frac{3x(x+2) + 3x + 6}{(2x-3)(x+2)}$

$= \frac{3x^2+6x+3x+6}{(2x-3)(x+2)}$

$= \frac{3x^2+9x+6}{(2x-3)(x+2)}$

$= \frac{3(x^2+3x+2)}{(2x-3)(x+2)}$

$= \frac{3(x+2)(x+1)}{(2x-3)(x+2)} = \frac{3(x+1)}{2x-3}$ if $x \neq -2$

(+2) Combines fractions appropriately
(+1) reduces
(+1) Domain restriction

5. (8 points) Use long division to divide $(2x^4 + 5x^3 - 15x^2 - 10x + 8) \div (x^2 - x - 2)$

$$
\begin{array}{r}
2x^2 + 7x - 4 \\
x^2 - x - 2 \enclose{longdiv}{2x^4 + 5x^3 - 15x^2 - 10x + 8} \\
-(2x^4 - 2x^3 - 4x^2) \\
\hline
7x^3 - 11x^2 - 10x \\
-(7x^3 - 7x^2 - 14x) \\
\hline
-4x^2 + 4x + 8 \\
-(-4x^2 + 4x + 8) \\
\hline
0
\end{array}
$$

(+2) setup
(+2) $2x^2$
(+2) $7x$
(+2) -4

6. (8 points) Write $-3x^2 - 24x - 43$ as the sum of a perfect square and a constant (i.e. change the $ax^2 + bx + c$ form of the expression into the $a(x-h)^2 + k$ form).

$-3x^2 - 24x - 43$

$-3(x^2 + 8x) - 43$ (+1)

$-3(x^2 + 8x + 16 - 16) - 43$ (+3)

$-3(x+4)^2 + 48 - 43$ (+3)

$-3(x+4)^2 + 5$ (+1)

7. (10 points) Given $f(x) = \frac{1-5x}{1+2x}$

(a) (5 points) Show f is a one-to-one function, using the definition of one-to-one.

Assume $f(a) = f(b)$ (+1)

$\frac{1-5a}{1+2a} = \frac{1-5b}{1+2b}$ (+1)

$(1-5a)(1+2b) = (1-5b)(1+2a)$ (+1)

$1 + 2b - 5a - 10ab = 1 + 2a - 5b - 10ab$

$2b - 5a = 2a - 5b$

$-7a = -7b$

$a = b$ ✓

(+2)

(b) (5 points) Find $f^{-1}(x)$, the inverse function of f.

$y = \frac{1-5x}{1+2x}$

$x = \frac{1-5y}{1+2y}$ (+2)

$x(1+2y) = 1-5y$

$x + 2xy = 1 - 5y$

$2xy + 5y = 1 - x$

$y(2x+5) = 1 - x$

$y = \frac{1-x}{2x+5}$ (+2)

$f^{-1}(x) = \frac{1-x}{2x+5}$

(+1)

Math 108: Exam 3 Name KEY

Fall 2013 Instructor ____________________

Page	1	2	3	4	5	Total	Course Points
Points	22	20	24	32	18	116	150
Score							

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1. (8 pts.) Find the solution to the following system of equations (use any legitimate algebraic method):

$$\begin{cases} 5x - 2y = 1 \\ y + 3 = -x \end{cases}$$

(2) $\Rightarrow y = -x - 3$

substitute

$5x - 2(-x-3) = 1$

$5x + 2x + 6 = 1$

$7x + 6 = 1$

$7x = -5$

$x = -\frac{5}{7}$

$y = -\left(-\frac{5}{7}\right) - 3$

$= \frac{5}{7} - \frac{21}{7}$

$= -\frac{16}{7}$

$\left(-\frac{5}{7}, -\frac{16}{7}\right)$

(+4) eliminates a variable

(+2) finds one coor

(+2) finds second coor based on first

2. (14 pts.) Find the solution to the following system of inequalities. Show your solution using a graph. Algebraically find the coordinates of all vertices of the system. Mark these points on your graph.

$$\begin{cases} y \le x^2 - 1 \\ -x + y \le 1 \end{cases}$$

$y = x^2 - 1$ think, $x^2 - 1 = 0$

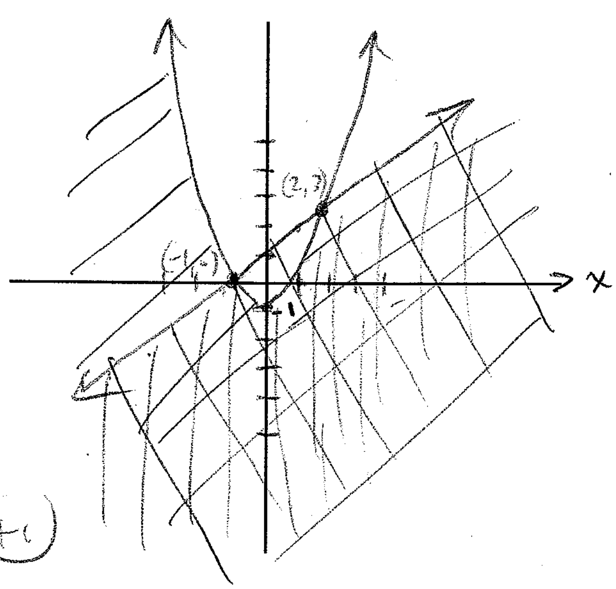
$x = \pm 1$

$y \le x + 1$ $y = x + 1$

(+2) Parabola

(+2) line

(+3) proper region shade



$$\begin{cases} y = x^2 - 1 \\ y = x + 1 \end{cases}$$

$x + 1 = x^2 - 1$ (+3)

$0 = x^2 - x - 2$

$0 = (x-2)(x+1)$

(+1) $x = 2$ $x = -1$ (+1)

$y = 2 + 1$ $y = -1 + 1$

(+1) $y = 3$ $y = 0$ (+1)

$(2, 3)$ $(-1, 0)$

$|x| \geq c \rightarrow$ $x \leq c$
$x \geq -c$

5. (24 pts.) Solve each inequality for x. Express your answers using <u>interval notation</u>.

6 (a) $|5 - 3x| \geq 8$

$5 - 3x \geq 8$ OR $5 - 3x \leq -8$ (+2) cases

$-3x \geq 3$ $\quad -3x \leq -13$ (+2) flips sign when dividing by negative

$x \leq -1$ $\quad x \geq \frac{13}{3}$

$(-\infty, -1] \cup [\frac{13}{3}, \infty)$ (+2) solution/int notation

6 (b) $|x - 5| < 7$ $\quad |x| < c \rightarrow -c < x < c$

$-7 < x - 5 < 7$

$+5 \quad +5 \quad +5$

$-2 < x < 12$

$(-2, 12)$

(+3) Double inequality
(+2) solution
(+1) interval notation

6 (c) $(x - 2)^2(x - 4)(x + 1) \geq 0$

Roots: 2, 4, −1

$+ \quad - \quad - \quad +$

$-1 \quad 2 \quad 4$

$(-\infty, -1] \cup \{2\} \cup [4, \infty)$

one-pt set

(+2) Identifies roots
(+2) testing of values
(+1) $(-\infty, -1] \cup [4, \infty)$
(+1) $\{2\}$

6 (d) $\frac{3x-5}{x+1} \leq 2$

$\frac{3x-5}{x+1} - 2 \leq 0$ (+1)

$\frac{3x-5}{x+1} - \frac{2(x+1)}{x+1} \leq 0$

$\frac{3x - 5 - 2x - 2}{x+1} \leq 0$

$\frac{x-7}{x+1} \leq 0$ (+2)

Look at zeroes of numerator <u>and</u> denominator

$+ \quad - \quad +$ (+2)

$-1 \quad 7$

$(-1, 7]$ (+1)

(+0) if student "cross multiplies" with no consideration of cases

9. (18 pts.) Solve each of the following equations for x.

4 (a) $6^x + 10 = 47$

$6^x = 37$ (+1)

$x = \log_6 37$ (+3)

4 (b) $\left(\frac{1}{8}\right)^x = 2^{x+3}$

$(2^{-3})^x$

$a^m = a^n$

$m = n$

$2^{-3x} = 2^{x+3}$ (+1)

$-3x = x + 3$ (+2)

$-4x = 3$

$x = -3/4$ (+1)

5 (c) $\log 5x + \log(x-1) = 2$

$\log(5x(x-1)) = 2$ (+1)

def $5x(x-1) = 10^2$ (+2)

$5x^2 - 5x - 100 = 0$

$5(x^2 - x - 20) = 0$

$5(x-5)(x+4) = 0$

$\boxed{x=5}$ (+1) $\cancel{x = -4}$ reject (+1)

5 (d) $5^x = e^{2x+1}$ take ln

$\ln 5^x = \ln e^{2x+1}$ (+1)

$x \ln 5 = 2x + 1$ (+2)

$x\ln 5 - 2x = 1$

$x(\ln 5 - 2) = 1$

$x = \dfrac{1}{\ln 5 - 2}$ (+2)

$\dfrac{\log e}{\log 5 - 2\log e} = x$

Test 4

Name KEY

1. (4 points) Determine two coterminal angles (one positive and one negative) for each angle below:

a) -130°

$-130^\circ + 360^\circ = 230^\circ$

$-130^\circ - 360^\circ = -490^\circ$

(+1) each
All or nothing

b) $\frac{11\pi}{6}$

$\frac{11\pi}{6} + 2\pi = \frac{11\pi}{6} + \frac{12\pi}{6} = \frac{23\pi}{6}$

$\frac{11\pi}{6} - 2\pi = \frac{11\pi}{6} - \frac{12\pi}{6} = -\frac{\pi}{6}$

2. (4 points) Find the complement and supplement for each angle.

a) 57°

Complement = 33° (+1)

Supplement = 123° (+1)

b) $\frac{2\pi}{5}$

Complement = $\frac{\pi}{10}$ (+1)

Supplement = $\frac{3\pi}{5}$ (+1)

if they answer any of the radian angles with a degree measure - take off 1 point total for not following directions.

3. (4 points) Change the units of measure.

a) Change 115° to radians.

$115^\circ \cdot \frac{\pi^r}{180^\circ} = \frac{115\pi}{180} = \frac{23\pi}{36}$

(+1) multiplies by $\frac{\pi}{180}$
(+1) simplifies.

b) Change 4^r to degrees.

$4^r \cdot \frac{180^\circ}{\pi^r} = \frac{720}{\pi}^\circ$

(+1) multiplies by $\frac{180}{\pi}$
(+1) simplifies.

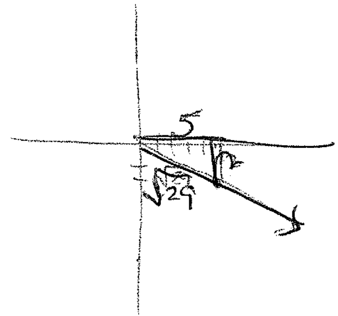
4. (16 points) Angle θ (in standard position) has terminal side that goes through $(5, -2)$.

a) What are the six trigonometric values for θ?

$\sin(\theta) = \frac{-2}{\sqrt{29}}$ (+2) $\sec(\theta) = \frac{\sqrt{29}}{5}$ (+2)

$\cos(\theta) = \frac{5}{\sqrt{29}}$ (+2) $\csc(\theta) = -\frac{\sqrt{29}}{2}$ (+2)

$\tan(\theta) = \frac{-2}{5}$ (+2) $\cot(\theta) = -\frac{5}{2}$ (+2)



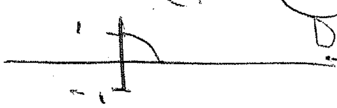
b) What are the coordinates of the point where the terminal side of θ intersects the unit circle?

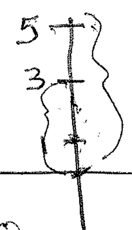
$\left(\frac{5}{\sqrt{29}}, \frac{-2}{\sqrt{29}} \right)$ (+4) – (+2) if reversed

5. (8 points) Give the equation of a cosine function $f(x)$ having all of the following properties:

- Its range is $[1, 5]$ $\frac{max - min}{2} = 2$
- Its period is 8π
- It has been reflected through the x-axis

$f(x) = -2\cos\left(\frac{1}{4}x\right) + 3$





(+2) Coefficient of 2
(+2) Coefficient on x of $\frac{1}{4}$
(+2) Vertical shift up 3
(+2) negated

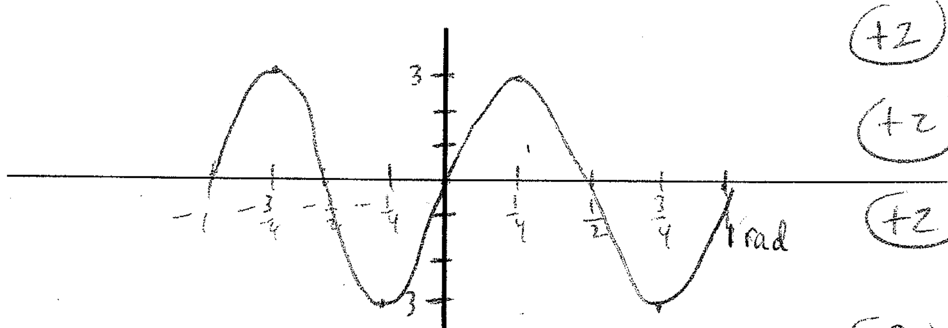
6. (8 points) Given $f(x) = 3\sin(2\pi x)$

$\sin Bx$ $P = \frac{2\pi}{B} = 1$

a) What is the amplitude of f? 3 (+1)

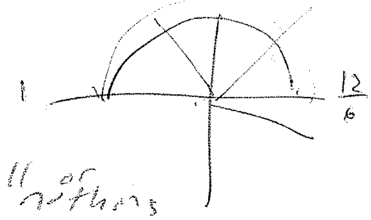
b) What is the period of f? $\frac{2\pi}{2\pi} = 1$ (+1)

c) Sketch the graph of f over at least two full periods on the axes below.



(+2) Correct amplitude / Range.
(+2) Has period of 1.
(+2) looks like sine.

(−2) if two full periods not given.



7. (16 points) Evaluate each of the following:

$\cos(\pi) =$ -1 (+2) all or nothing

$\cos\left(\frac{11\pi}{6}\right) =$ $\frac{\sqrt{3}}{2}$ (+2) all or nothing

$\sin\left(\frac{\pi}{3}\right) =$ $\frac{\sqrt{3}}{2}$ (+2) all or nothing

$\sin\left(\frac{5\pi}{4}\right) =$ $-\frac{\sqrt{2}}{2}$ (+2) all or nothing

4, 2, 3, 6 $\frac{\pi}{4}, \frac{\pi}{2}, \frac{\pi}{3}, \frac{\pi}{6}$

$\tan\left(\frac{2\pi}{3}\right) =$ $-\sqrt{3}$ (+2) all or nothing

$\sin\left(\frac{5\pi}{12}\right) =$ $\sin\left(\frac{3\pi}{12}+\frac{2\pi}{12}\right)$

$= \sin\left(\frac{\pi}{4}\right)\cos\left(\frac{\pi}{6}\right) + \cos\left(\frac{\pi}{4}\right)\sin\left(\frac{\pi}{6}\right)$

$= \frac{\sqrt{2}}{2}\cdot\frac{\sqrt{3}}{2} + \frac{\sqrt{2}}{2}\cdot\frac{1}{2}$

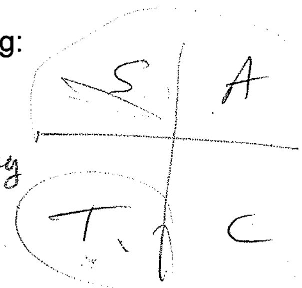
$= \frac{\sqrt{6}+\sqrt{2}}{4}$

(+2) identifies to use a formula

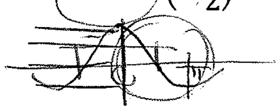
(+2) applies it correctly

(+2) answer (be mindful of other forms)

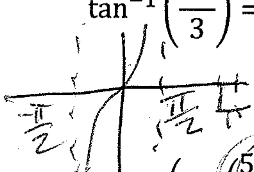
8. (15 points) Evaluate each of the following:



$\arccos\left(-\frac{1}{2}\right) =$ $\frac{2\pi}{3}$ (+3) all or nothing



$\tan^{-1}\left(\frac{\sqrt{3}}{3}\right) =$ $\frac{\pi}{6}$ (+3) all or nothing

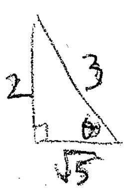


$\arcsin\left(\sin\left(\frac{5\pi}{4}\right)\right) =$ $-\frac{\pi}{4}$

(+1) Computes $\sin\left(\frac{5\pi}{4}\right)$ (does not need to be shown)

(+2) answer

$\boxed{f^{-1}(f(x)) = x}$



$\cos\left(\sin^{-1}\left(\frac{2}{3}\right)\right) =$ $\frac{\sqrt{5}}{3}$

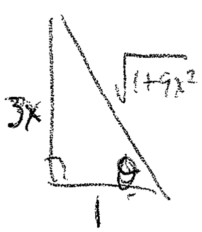
(+1) draws Δ with sine of some angle $\frac{2}{3}$.

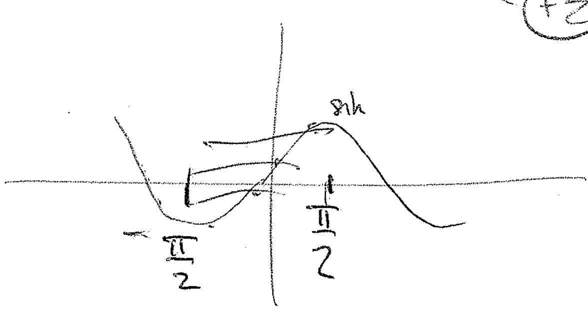
(+2) answer

$\sec(\arctan(\frac{3x}{1})) =$ $\sqrt{1+9x^2}$

(+1) draws Δ with tangent of some angle $3x$.

(+2) answer.





10. (8 points) Verify the trigonometric identity:

$$\frac{\sin(\theta)}{1+\cos(\theta)} + \frac{1+\cos(\theta)}{\sin(\theta)} = 2\csc(\theta)$$

LCD

(+3) Combines fractions
(+3) Uses Pythagorean Identity
(+2) Uses reciprocal identity

$$\frac{\sin(\theta)\sin(\theta)}{(1+\cos(\theta))\sin(\theta)} + \frac{(1+\cos(\theta))(1+\cos(\theta))}{(1+\cos(\theta))\sin(\theta)}$$

$$= \frac{\sin^2(\theta) + 1 + 2\cos(\theta) + \cos^2(\theta)}{(1+\cos(\theta))\sin(\theta)}$$ Identity

factor

Combine like terms $\quad \frac{2 + 2\cos(\theta)}{(1+\cos(\theta))\sin(\theta)} = \frac{2\cancel{(1+\cos(\theta))}}{\cancel{(1+\cos(\theta))}\sin(\theta)} = \frac{2}{\sin(\theta)} = 2\csc(\theta)$

$\frac{2}{\sin\theta}$

11. (8 points) Verify the trigonometric identity:

$$\frac{\cos(\alpha-\beta)}{\sin(\alpha)\sin(\beta)} = \cot(\alpha)\cot(\beta) + 1$$

$\cos(\alpha \pm \beta) = \cos\alpha\cos\beta \mp \sin\alpha\sin\beta$

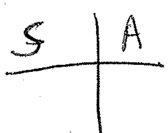
$$\frac{\cos(\alpha-\beta)}{\sin(\alpha)\sin(\beta)} = \frac{\cos(\alpha)\cos(\beta)+\sin(\alpha)\sin(\beta)}{\sin(\alpha)\sin(\beta)} \quad (+4)$$

$\frac{A+B}{C} = \frac{A}{C} + \frac{B}{C}$

$$= \frac{\cos(\alpha)\cos(\beta)}{\sin(\alpha)\sin(\beta)} + \frac{\cancel{\sin(\alpha)\sin(\beta)}}{\cancel{\sin(\alpha)\sin(\beta)}} \quad (+2)$$

$$= \cot(\alpha)\cot(\beta) + 1 \quad (+2)$$

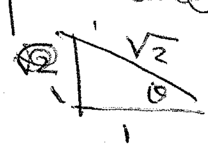
12. (8 points) Solve for x in the following equation. Find ALL solutions.



$2\cos(5x) = -\sqrt{2}$

$\cos(5x) = -\frac{\sqrt{2}}{2}$

(+2) isolates $\cos(5x)$



$5x = \frac{3\pi}{4} + 2\pi n$

$x = \frac{3\pi}{20} + \frac{2\pi}{5}n$

(+2) Identifies $\frac{3\pi}{4}$

$5x = \frac{5\pi}{4} + 2\pi n$

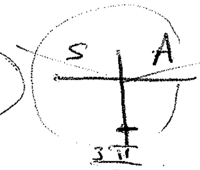
$x = \frac{5\pi}{20} + \frac{2\pi}{5}n$

(+2) Identifies $\frac{5\pi}{4}$

(−2) if no coterminals

(+2) Divides by 5.

13. (8 points) Solve for x in the following equation. Find solutions on the interval $[0, 2\pi)$



$1 + \sin(x) = 2\cos^2(x)$

$1 + \sin(x) = 2(1 - \sin^2(x))$

$1 + \sin(x) = 2 - 2\sin^2(x)$

$2\sin^2(x) + \sin(x) - 1 = 0$

factor $(2\sin(x) - 1)(\sin(x) + 1) = 0$

$2\sin x - 1 = 0$

$\sin(x) = \frac{1}{2}$

$x = \frac{\pi}{6}$ (+1)

$x = \frac{5\pi}{6}$ (+1)

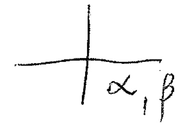
$\sin(x) = -1$

$x = \frac{3\pi}{2}$ (+1)

(+2) uses Pythagorean identity
(+2) factors "quadratic"
(+1) sets factors to zero

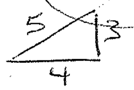
$AB = 0$

$A = 0 \quad B = 0$



14. (20 points) Suppose $\sin(\alpha) = -\frac{5}{13}$ and $\cos(\beta) = \frac{4}{5}$ such that $\boxed{\frac{3\pi}{2} < \alpha < 2\pi}$ and $\frac{3\pi}{2} < \beta < 2\pi$

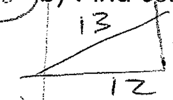
+2 a) Find $\sin(\beta)$.



$\sin(\beta) = -\frac{3}{5}$

{ (+1) Draws triangle, finds 3rd side (not necessary)
{ (+1) Answer (including −)

+2 b) Find $\cos(\alpha)$.



$\cos(\alpha) = \frac{12}{13}$

{ (+1) Draws triangle, finds 3rd side (not nec)
{ (+1) Answer (including +)

+3 c) Find $\cos(\alpha + \beta)$. $= \cos(\alpha)\cos(\beta) - \sin(\alpha)\sin(\beta)$

identity $= \underbrace{\left(\frac{12}{13}\right)\left(\frac{4}{5}\right) - \left(\frac{-5}{13}\right)\left(\frac{-3}{5}\right)}_{+2} = \frac{48}{65} - \frac{15}{65} = \frac{33}{65}$ (+1)

+3 d) What quadrant is $\alpha + \beta$ in? Justify your answer.

$\underbrace{\cos(\alpha+\beta) > 0}_{+1}$ so Quadrant 4 (+1)

$\frac{3\pi}{2} < \alpha < 2\pi$
$\frac{3\pi}{2} < \beta < 2\pi$ $\Rightarrow$ $\alpha+\beta$ is in Q3 or Q4. } (+1)
$\Rightarrow 3\pi < \alpha + \beta < 4\pi$

+3 e) Find $\sin(2\alpha)$.

$\sin(2\alpha) = 2\sin(\alpha)\cos(\alpha) = \underbrace{2\left(-\frac{5}{13}\right)\left(\frac{12}{13}\right)}_{+2} = \frac{-120}{169}$ (+1)

+3 f) What quadrant is 2α in? Justify your answer.

$\boxed{\begin{array}{l}\frac{3\pi}{2} < \alpha < 2\pi \\ 3\pi < 2\alpha < 4\pi\end{array}}$

$\Rightarrow 2\alpha$ is in Q3 or Q4 (+1)

$\cos(2\alpha) = \cos^2(\alpha) - \sin^2(\alpha)$
$= \left(\frac{12}{13}\right)^2 - \left(-\frac{5}{13}\right)^2$
$= \frac{144}{169} - \frac{25}{169} = \frac{119}{169} > 0$ (+1)
$\Rightarrow$ Q4. (+1)

+2 g) Find $\cos\left(\frac{\beta}{2}\right)$.

$\cos\left(\frac{\beta}{2}\right) = -\sqrt{\frac{1+\cos(\beta)}{2}} = -\sqrt{\frac{1+4/5}{2}}$

(+1) (+1)

+2 h) What quadrant is $\frac{\beta}{2}$ in? Justify your answer.

$\frac{3\pi}{2} < \beta < 2\pi$

$\frac{3\pi}{4} < \frac{\beta}{2} < \pi$ (+1) $\Rightarrow$ $\frac{\beta}{2}$ is in Quadrant 2. (+1)