Sec 5.3 HW Solutions

#1) Factoring (review of Ch. 2, but now we solve for the roots of the fcn. $f(x)$)

a) $x^2 - 2x - 15 = 0$

$(x-5)(x+3) = 0$

$x = 5, -3$

b) $2x^2 - x = 1$

$2x^2 - x - 1 = 0$

$(2x+1)(x-1) = 0$

$x = -1/2, 1$

c) $-x^2 - x + 12 = 0$

$-(x^2 + x - 12) = 0$

$-(x-3)(x+4) = 0$

$x = 3, -4$

d) $8x^2 + 14x - 4 = 0$

$4x^2 + 7x - 2 = 0$

$(4x-1)(x+2) = 0$

$x = 1/4, -2$

e) $x^2 - 5 = 0$

$(x+\sqrt{5})(x-\sqrt{5}) = 0$

$x = \pm\sqrt{5}$

(A difference of 'squares' though not perfect squares)

$\longrightarrow$ Before, we'd set $x^2 = $ constant and solve, as in

$\sqrt{x^2} = \sqrt{5}$

$x = \pm\sqrt{5}$

$\Rightarrow$ The right side method is still fine, but in the context of factoring as $(x-r_1)(x-r_2)$, we use the left side method.

#2 a) $9x^2 - 16 = 0$

$9x^2 = 16$

$x^2 = 16/9$

$x = \pm\sqrt{16/9} = \pm 4/3$

b) $2x^2 - 6x = -3$

$2(x^2 - 3x) = -3$

$2(x^2 - 3x + (-3/2)^2) = -3 + 2\cdot(-3/2)^2$

$(x - 3/2)^2 = \dfrac{-3 + 9/2}{2}$

$(x - 3/2)^2 = \dfrac{3}{4} \longrightarrow$

#6) The number of roots a quadratic equation has is determined as follows:

i) $b^2 - 4ac > 0 \Rightarrow 2$ roots

ii) $b^2 - 4ac = 0 \Rightarrow$ 1 root with multiplicity 2

iii) $b^2 - 4ac < 0 \Rightarrow$ no roots

We sometimes refer to situation (ii) as a double root.

#7) $f(x) = x^2 + 8x + t$ will have 2 solns (roots) if the discriminant is positive:

$$-8 - 4t > 0, \text{ that is, } t < -2.$$

It follows that $t = -2$ gives one root and $t > -2$ gives no soln.

#8) This time, the discriminant is $t^2 + 80$:

i) $t^2 + 80 > 0$ gives 2 solns.

That is, $t^2 > -80$, which is true for all t.

(ii + iii don't occur since

$t^2 + 80 = 0$ has no solution,

nor does $t^2 + 80 < 0$.)

#9) Symmetry of the parabola gives the answer: