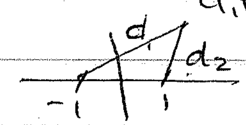
Ellipses – a summary

(1) An example was given from which computation of the distances from a pt (x, y) on the ellipse to a known focus $(c, 0)$, and to the other focus $(-c, 0)$, were found by Pyth. thm.

It was noted that the sum of these distances is the fixed sum $D = d_1 + d_2$ for the example. (p. 303)

$d_1 + d_2 = 6$



(2) The algebra simplifying this computation led to an ~~simplified~~ equation describing the relationship between x and y for this ellipse, namely:

$$\frac{x^2}{(D/2)^2} + \frac{y^2}{(D/2)^2 - c^2} = 1$$

In the example, $D = 6$, $c = 1$, so the eqn. was

$$\cancel{\frac{x^2}{(6/2)^2} + \frac{y^2}{(6/2)^2 - 1^2} = 1}$$

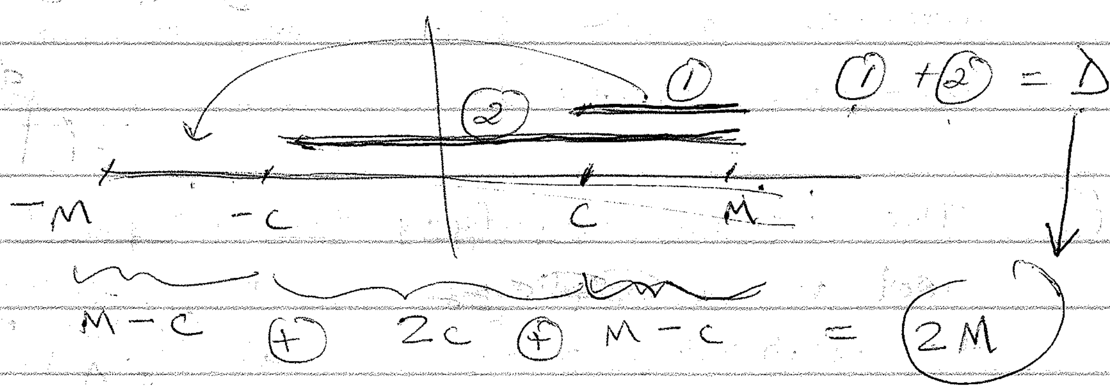
Ex $$\boxed{\frac{x^2}{9} + \frac{y^2}{8} = 1}$$

(3) The example served to model an eqn. for any ellipse w/ foci $(-c, 0)$, $(c, 0)$ and fixed distance sum D.

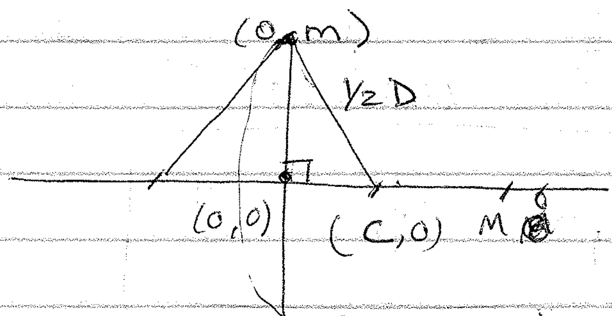
That is, $\dfrac{x^2}{(D/2)^2} + \dfrac{y^2}{\underbrace{(D/2)^2 - c^2}_{m^2}} = 1$

(4) The length of the axis on which the foci lie we named $2M$, where M is the distance from the origin to the vertex.

By geometric construction and by algebra, we saw that $2M = D$



(5) Finally, we call the distance of the shorter axis $2m$, with m being the ~~side~~ vertical of the right triangle with hypotenuse $D/2$ and base c



Thus, $m^2 + c^2 = (\tfrac{1}{2}D)^2 = M^2$

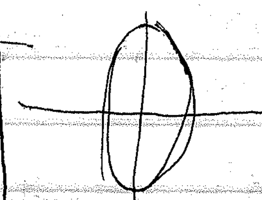
or $m^2 = M^2 - c^2$

Hence, the ellipse centered on $(0,0)$ has eqn $\dfrac{x^2}{M^2} + \dfrac{y^2}{\underbrace{M^2 - c^2}_{m^2}} = 1$

~~Corollary~~ (if the major axis is on the y-axis ~~y-axis holds the major~~

If the foci are on the vertical, this eqn: becomes

$$\boxed{\frac{x^2}{m^2} + \frac{y^2}{M^2} = 1}$$



Just remember, the foci are on the major (longer) axis by convention.

Problems - Sec 9.2

① a) $\frac{x^2}{100} + \frac{y^2}{64} = 1$ $\quad 100 = 10^2, \quad 64 = 8^2$

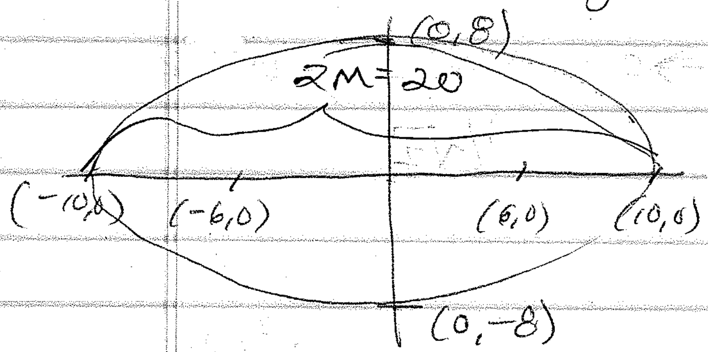
$M^2 = 100, \quad M = 10$

$m^2 = 64, \quad m = 8$

$m^2 = M^2 - c^2$

$c^2 = M^2 - m^2 = 100 - 64$

$c^2 = 36 \longrightarrow c = 6$



$D = 2M = 2 \cdot 10 = 20$

fixed sum

b) $\frac{x^2}{4} + \frac{y^2}{5} = 1$

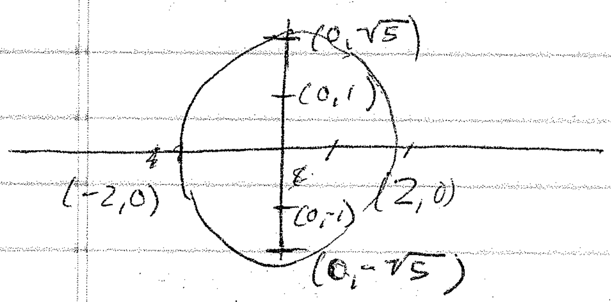
$M^2 = 5, \quad M = \sqrt{5}$

$m^2 = 4, \quad m = 2$

$c^2 = M^2 - m^2 = 5 - 4 = 1$

$c = 1, \quad D = 2M = 2\sqrt{5}$

fixed sum



c) $4x^2 + 9y^2 = 36$

$\frac{x^2}{9} + \frac{y^2}{4} = 1 \longrightarrow$

$M^2 = 9, \quad M = 3$

$m^2 = 4, \quad m = 2$

$c^2 = M^2 - m^2 = 9 - 4 = 5$

$c = \sqrt{5}, \quad D = 2 \cdot 3 = 6$

"fixed sum"

d) $9x^2 + 25y^2 = 225$

$$\frac{9x^2}{225} + \frac{25y^2}{225} = 1 \qquad m^2 = 9,\ M^2 = 25$$

$$\frac{x^2}{225/9} + \frac{y^2}{225/25} = 1 \qquad m = 3,\ M = 5$$

$$c^2 = 25 - 9 = 16$$

$$c = 4$$

$$\frac{x^2}{(15/3)^2} + \frac{y^2}{9} = 1 \qquad D = 2M = 10$$

↖25 :)

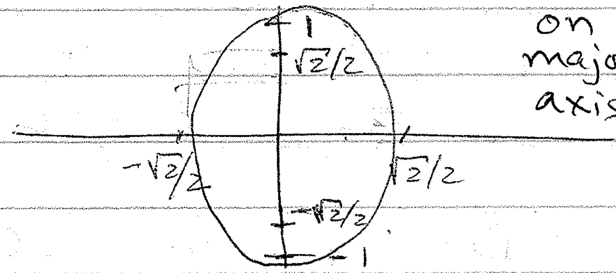
e) $2x^2 + y^2 = 1$ $\qquad m^2 = 1/2 \quad M^2 = 1$

$\rightarrow m = \sqrt{2}/2 \quad M = 1$

$$\frac{x^2}{1/2} + \frac{y^2}{1} = 1 \qquad c^2 = 1 - 1/2 = 1/2$$

on major axis $\rightarrow c = \sqrt{2}/2$, $D = 2$

$1/\sqrt{2}$

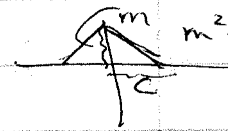


#2 a) $c = 4$, $D = 10 = 2M$, $M = 5$, $M^2 = 25$, $m = ?$

$$m^2 = M^2 - c^2 = 25 - 16 = 9, \quad m = 3$$

$$\frac{x^2}{25} + \frac{y^2}{9} = 1$$

b) $c = 5$, $D = 20 = 2M$, $M = 10$, $M^2 = 100$



$m^2 + c^2 = M^2$

$$m^2 = M^2 - c^2 = 100 - 25 = 75$$

from $m^2 + c^2 = M^2$

$$\frac{x^2}{75} + \frac{y^2}{100} = 1$$

#2c) $(0,8), (0,-8)$ vertices, $2m = 6$

$c = 8, \quad m = 3, \quad M^2 = m^2 + c^2$

$c^2 = 64, \quad m^2 = 9 \qquad = 9 + 64$

$= 73$

$$\frac{x^2}{9} + \frac{y^2}{73} = 1$$

d) $(-13,0), (13,0) \longrightarrow$ ~~celse~~, $2M = 26, \ M = 13$

$(0,9), (0,-9) \longrightarrow 2m = 18, \ m = 9$

$m^2 + c^2 = M^2 \longrightarrow 9^2 + c^2 = 13^2$

$c^2 = 169 - 81 = 88$

$c = \sqrt{88} = 2\sqrt{22}$

$$\frac{x^2}{169} + \frac{y^2}{81} = 1$$

#3a) $\dfrac{(x+5)^2}{16} + \dfrac{(y-1)^2}{9} = 1$ Center $(-5,1)$

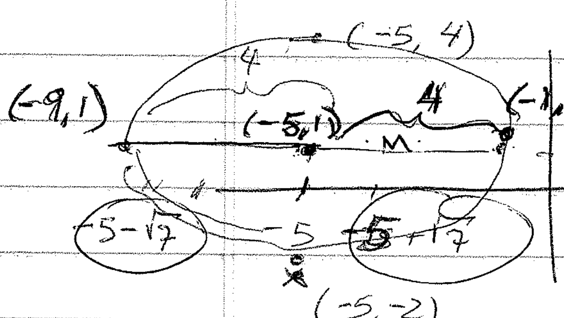
$M^2 = 16 \rightarrow M = 4$

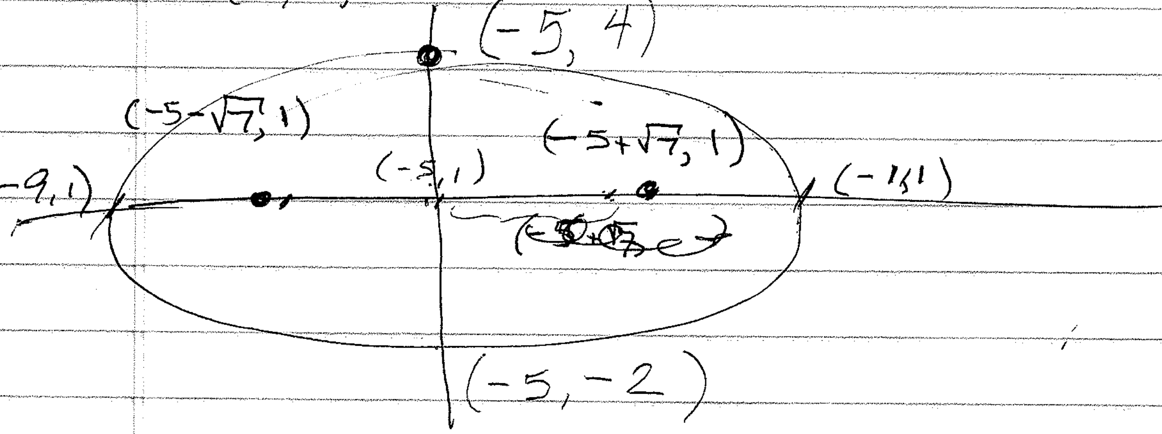
$m^2 = 9 \rightarrow m = 3$

Pyth th. $\rightarrow m^2 + c^2 = M^2 \rightarrow c^2 = M^2 - m^2 = 16 - 9$

length of focal dist $\rightarrow c = \sqrt{7}$

$D = 2M = 8$





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