Ch. 2.4

#7e) $$\sum_{i=0}^{n} (i+1)(i+2) = \ ?$$

notice that this has an extra term over #7d.

#7d) $$\sum_{i=1}^{n} (i+1)(i+2) = \frac{n^3 + 6n^2 + 11n}{3}$$

(my work not shown; expand $(i+1)(i+2)$ + apply special sums formulas)

$$\sum_{i=0}^{n} (i+1)(i+2) = \underbrace{(0+1)(0+2)}_{\substack{i=0\\ \text{term}}} + \sum_{i=1}^{n} (i+1)(i+2)$$

$$= 2 + \sum_{i=1}^{n} (i+1)(i+2)$$

Ans to #7e $$= 2 + \frac{n^3 + 6n^2 + 11n}{3}$$

Ch. 7.2

#6a) Simplify $\dfrac{|x+2|}{x+2}$,

Case 1 Clearly $\dfrac{x+2}{x+2} = 1 \quad (x \neq -2)$

This is the answer when $x+2 > 0$

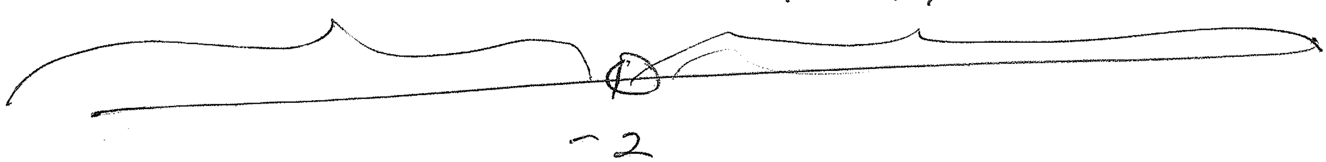
or $x > -2$

Case 2 $\dfrac{-(x+2)}{x+2} = -1 \quad (x \neq -2)$

This is the answer when $x+2 < 0$

or $x < -2$

$|x+2|/x+2 = -1$ $\qquad\qquad$ $|x+2|/x+2 = 1$



$$\frac{|x+2|}{x+2} = \begin{cases} 1, & x > -2 \\ -1, & x < -2 \end{cases}$$

#6b) Simplify $\dfrac{|x^2-4|}{|3x+2|}$

$$\frac{|x^2-4|}{|3x+6|} = \frac{|(x-2)\cdot(x+2)|}{|3\cdot(x+2)|} = \frac{|x-2|\,\cancel{|x+2|}}{3\,\cancel{|x+2|}}$$

$$= \frac{|x-2|}{3} = \begin{cases} \dfrac{x-2}{3}, & x-2 \geq 0 \text{ or } x \geq 2 \\ -\dfrac{(x-2)}{3}, & x-2<0 \text{ or } x<2 \end{cases}$$

as long as $x \neq -2$ from original domain

#7f) $\sum_{i=4}^{n} (i^2 - i + 3)$ notice $i = 4$, not $\underline{1}$

Use the idea that $\sum_{i=m}^{n} a_i = \sum_{i=1}^{n} a_i - \sum_{i=1}^{m-1} a_i$ ✱

For example, $m = 4$ is start.

$n = 20$ is end.

$$\sum_{m=4}^{20} a_i = \sum_{i=1}^{20} a_i - \sum_{i=1}^{3} a_i \qquad 3 \to m-1 = 4-1$$

$\underline{\underline{Ex}}$ $\sum_{i=1}^{n} i = \frac{n(n+1)}{2}$ sum of first n integers

$\sum_{m=4}^{20} i = 4 + 5 + 6 + \cdots + 20$ but we don't have a "closed form" solution when $i \neq 1$

We could add longhand $\underline{\underline{or}}$ apply ✱

and $\sum_{i=1}^{n} i = \frac{n(n+1)}{2}$

$$\sum_{i=4}^{20} i = \sum_{i=1}^{20} i - \sum_{i=1}^{3} i = \frac{20(21)}{2} - \frac{3(4)}{2} \text{ etc.,}$$

Do # 7f like this.

$$\sum_{i=4}^{n} (i^2 - i + 3) = \sum_{i=1}^{n} (i^2 - i + 3) - \sum_{i=1}^{3} (i^2 - i + 3)$$

$$= \frac{n(n+1)(2n+1)}{6} - \frac{n(n+1)}{2} - 3n$$

$$- \left(\frac{3(3+1)(2\cdot 3+1)}{6} - \frac{3(3+1)}{2} - 3\cdot 3 \right)$$

$$= \frac{n(n+1)(2n+1) - 3n(n+1) - 18n}{6} - (-1)$$

first sum

second sum

etc

Leave the answer in terms of n but simplified

$$= \frac{n^3 + 8n - 51}{3}$$

#8b)

$$\sum_{i=4}^{79}\left(\frac{1}{i+1}-\frac{1}{i}\right) = \sum_{i=4}^{79}\frac{1}{i+1} - \sum_{i=4}^{79}\frac{1}{i}$$

$$= \left(\cancel{\frac{1}{5}}+\cancel{\frac{1}{6}}+\cancel{\frac{1}{7}}+\cdots\cancel{\frac{1}{79}}+\frac{1}{80}\right)$$

$$-\left(\frac{1}{4}+\cancel{\frac{1}{5}}+\cancel{\frac{1}{6}}+\cancel{\cdots}+\cancel{\frac{1}{79}}\right)$$

$$= \frac{1}{80}-\frac{1}{4} = -\frac{19}{80}$$

8 Telescoping sums

Telescoping sums are of the form in this problem $\sum_{i=1}^{n} (a_i - a_{i+1})$

& when you expand this, a lot of terms cancel!

Note, on these you generally do not have a "closed form soln" (like $\frac{n(n+1)}{2}$) but when you expand the terms, you'll see the pattern.

8a) $\sum_{i=1}^{50} (3^i - 3^{i+1}) =$ break it up

$= \sum_{i=1}^{50} 3^i - \sum_{i=1}^{50} 3^{i+1} = 3^1 + 3^2 + 3^3 + \cdots + 3^{49} + 3^{50}$
$- (3^2 + 3^3 + 3^4 + \cdots + 3^{50} + 3^{51})$

$= 3^1 + \cancel{3^2} + \cancel{\cdots} + \cancel{3^{50}} - \cancel{3^2} - \cancel{3^3} - \cdots - \cancel{3^{50}} - 3^{51}$

$= 3^1 - 3^{51}$ done