Sec 5.1 HW Solutions

#1. $P(x) = a_n x^n + a_{n-1} x^{n-1} + \dots + a_2 x^2 + a_1 x + a_0$

where the a_i are coefficients

+ the n are non-negative integers.

So ... only a, c, e, g are polynomials.

note e) 4 is the polynomial with all coeffs = 0 except a_0, which is 4.

$P(x) = 4$ is a horizontal line through $y = 4$.

#3) The y-intercept is always $P(0)$.

$P(0) = a_0$ but you don't need to multiply out the terms to compute this. Just substitute $x = 0$ into the given equation.

a) $f(0) = (-2)^3(-1) = 8$

b) $f(0) = 0(0+1)^4(0+3) = 0$

c) $f(0) = -(0+1)(0-2)(0+3) = 6$

d) $f(0) = (1)(4)(0) = 0$

e) $f(0) = (3)^2(2)^2(-1)^5 = -36$

f) $f(0) = 2^2 \cdot 3^2 \cdot 1^4 = 36$

The leading coeff is a_n, but you don't need to multiply out the terms to find it. Just multiply the coeffs of factored x terms.

However, if a factor is raised to a power other than 1, adjust the coeff. accordingly:

a) $(x-2)^3(x-1)^1 \longrightarrow 1^3 \cdot 1^1 = 1$

b) $x(x+1)^4(x+3)^1 \longrightarrow 1 \cdot 1^4 \cdot 1 = 1$

c) $-(x+1)(x-2)(x+3) \longrightarrow -1 \cdot 1 \cdot 1 = -1$

d) $(2x+1)(-x+4)x^2 \longrightarrow 2 \cdot -1 \cdot 1 = -2$

e) $(3-x)^2(2-x)^2(x-1)^5 \rightarrow (-1)^2 \cdot (-1)^2 \cdot 1^5$
$= 1$

f) $(2-x)^2(3+x)^2(x+1)^4 = (-1)^2 \cdot 1^2 \cdot 1^4$
$= 1$

The ~~deg~~ degree is the sum of the exponents on the factors — again, no need to multiply out:

a) $n = 3+1 = 4$

b) $n = 1+4+1 = 6$

c) $n = 2+2+5 = 9$

d) $n = 1+1+2 = 4$

e) $n = 2+2+5 = 9$

f) $n = 2+2+4 = 8$

<u>Roots</u> (values of x where $f(x) = 0$; also called the x-intercepts or the "zeroes")

2.

a) $f(x) = (x-2)^3 (x-1) = 0$

~~Zeroes:~~ Roots: when $(x-2)^3 = 0$ or $x-1=0$

so $x-2=0$ $\quad x-1=0$

$x=2 \quad x=1$

"Multiplicity" refers to number of times a factor $x-r$ occurs in the factorization of a polynomial. The multiplicity (call it M) of the factor $(x-r)$ is also the mult of the root r itself.

So M of $r_1 = 2$ is 3

M of $r_2 = 1$ is 1.

Comment: A template for a fully factored polynomial is given as

$$(x-r_1)(x-r_2)\cdots(x-r_n)$$

where $r_1, r_2, \ldots, r_n$ are the roots. The minus sign is the convention; the signs of the individual roots may be (+) or (−) e.g.

$(x+2)(x+7)(x-1)$ has roots

$$r_1=-2,\ r_2=-7,\ r_3=1$$

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b) $f(x) = x^1(x+1)^4(x+3)^1$

Roots: $\left.\begin{array}{l} r_1 = 0, \quad M_1 = 1 \\ r_2 = -1, \quad M_2 = 4 \\ r_3 = -3, \quad M_3 = 1 \end{array}\right\}$ Multiplicities

c) $f(x) = -(x+1)^1(x-2)^1(x+3)^1$

Roots: $\left.\begin{array}{l} r_1 = -1, \quad M_1 = 1 \\ r_2 = 2, \quad M_2 = 1 \\ r_3 = -3, \quad M_3 = 1 \end{array}\right\}$ Multiplicities

d) $f(x) = (2x+1)^1(-x+4)^1 x^2$

Roots: $\left.\begin{array}{l} r_1 = -\frac{1}{2}, \quad M_1 = 1 \\ r_2 = 4, \quad M_2 = 1 \\ r_3 = 0, \quad M_3 = 2 \end{array}\right\}$ Multiplicities

e) $f(x) = (3-x)^2(2-x)^2(x-1)^5$

Roots $\left.\begin{array}{l} r_1 = 3, \quad M_1 = 2 \\ r_2 = 2, \quad M_2 = 2 \\ r_3 = 1, \quad M_3 = 5 \end{array}\right\}$ Multiplicities

f) $f(x) = (2-x)^2(3+x)^2(x+1)^4$

Roots $\left.\begin{array}{l} r_1 = 2, \quad M_1 = 2 \\ r_2 = -3, \quad M_2 = 2 \\ r_3 = -1, \quad M_3 = 4 \end{array}\right\}$ Multiplicities

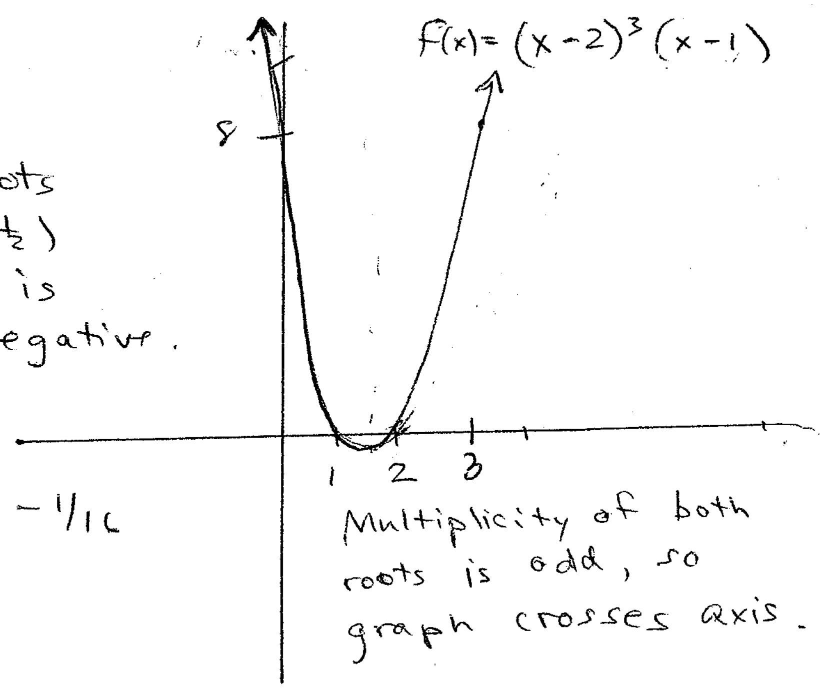
Sec 5.1 #3

Skitches: Besides the info above, look at shape of mother fcn, and sign of lead coeff. Also, if M is odd, the graph crosses the x-axis, and if M is even, it only touches it.

It's a good idea to plot a pt. or two between roots.

#3a)

$f(x) = (x-2)^3(x-1)$



Between roots (say $x = 1\frac{1}{2}$) the graph is slightly negative.

$f(3/2) = -1/16$

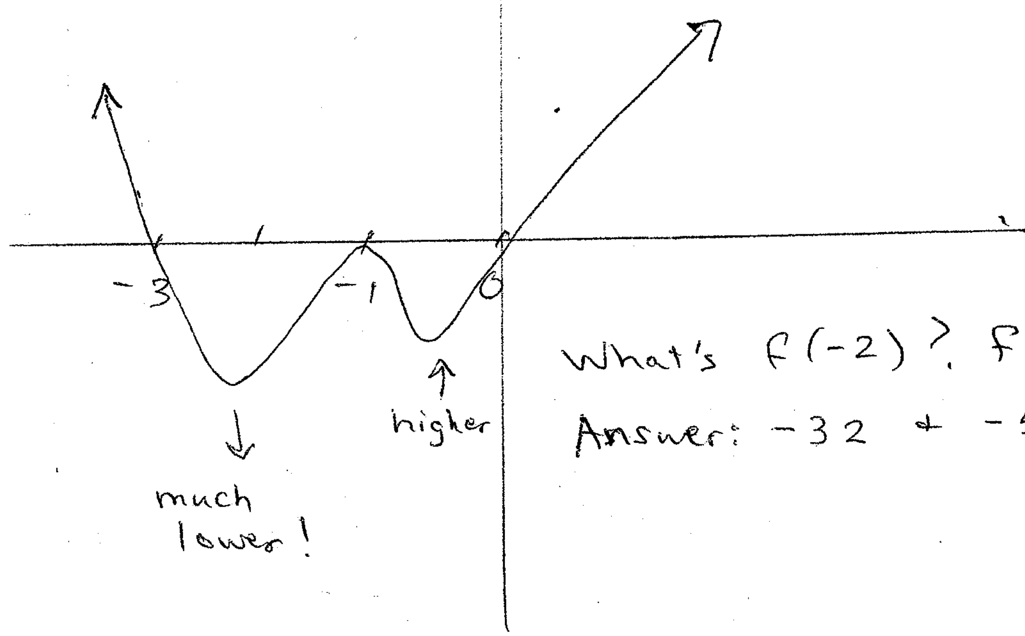
Multiplicity of both roots is odd, so graph crosses axis.

3 b) y-int = 0, roots: $-3, -1, 0$

M of -3 + 0 is one. (Crosses)

M of -1 is four. (Touches)

$y = x(x+1)^4(x+3)$

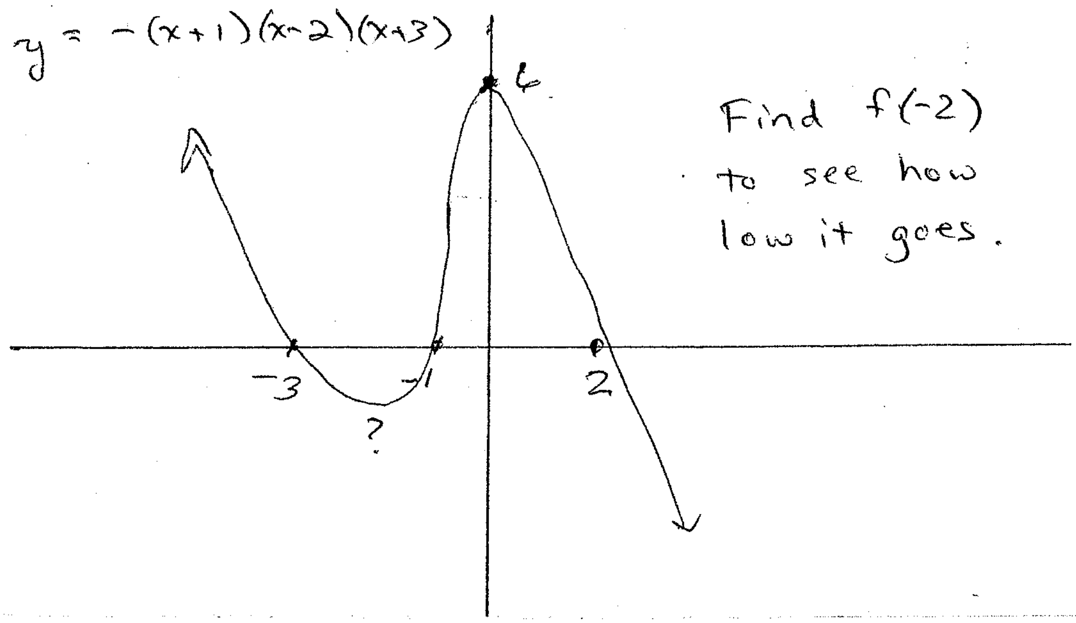


What's $f(-2)$? $f(-\frac{1}{2})$?

Answer: -32 + $-5/64$

3 c) y-int: 6 roots: $-1, 2, -3$ M=1 for all (crosses at ~~all roots~~)

$y = -(x+1)(x-2)(x+3)$



Find $f(-2)$ to see how low it goes.

#3d) y-int: 0 roots: $-1/2$, 4, 0

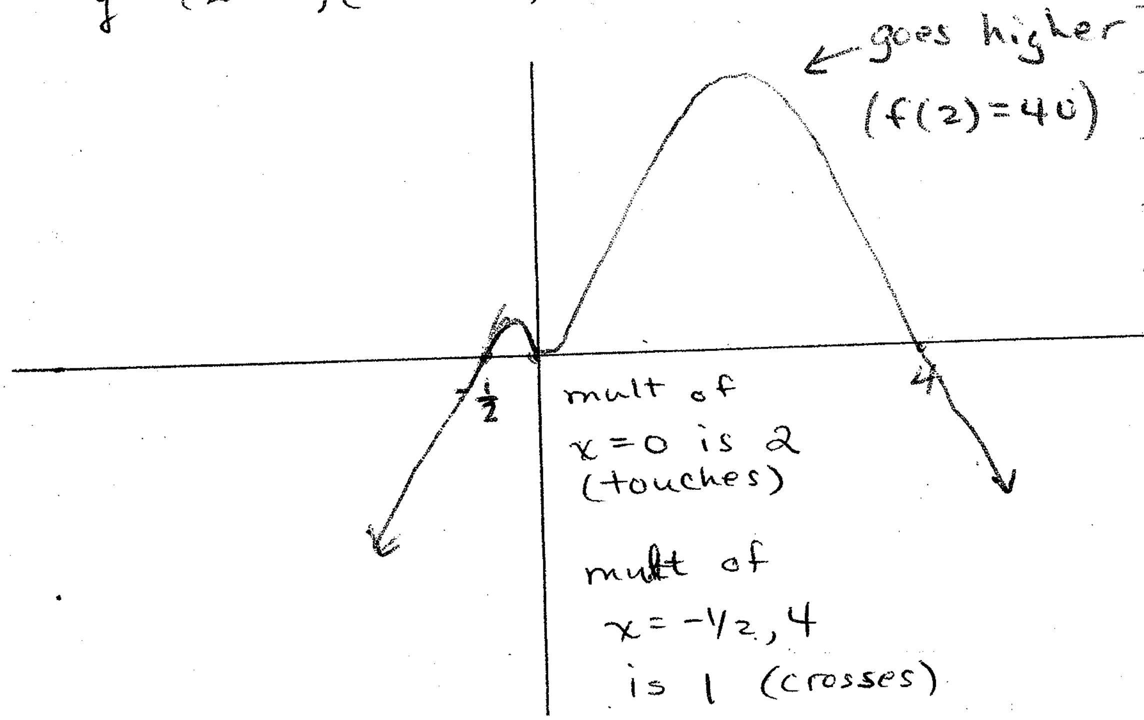
<u>Note</u>: The leading coeff works out to be negative:

$$y = (2x+1)(-x+4)(x^2)$$
$$= (-2x^2 + 7x + 4)x^2$$
$$= -2x^4 + 7x^3 + 4x^2$$

Test pt: $x = 2$ (halfway between roots $x=0$ + $x=4$)

$$y = (5)(2)(4) = 40$$

$$y = (2x+1)(-x+4)x^2$$



#3e) y-int: −36   roots: $\underbrace{3, 2}_{\text{even}}, \underbrace{1}_{\text{odd}}$

M :

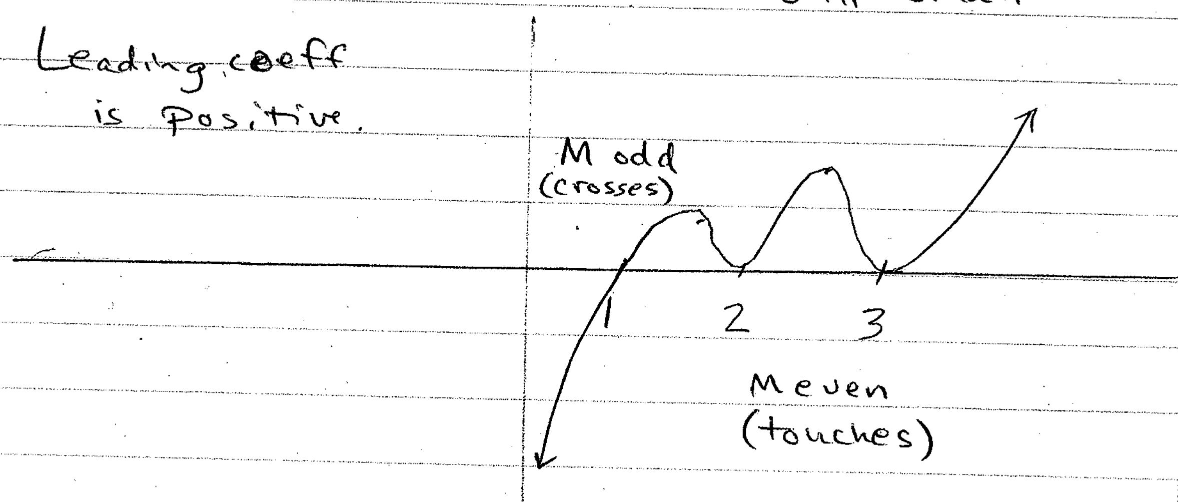
We can't show y-int on graph w/o scaling this other than 1:1. But we ought to inspect values of y between the roots x = 1, 2, 3

$$f(3/2) = (3-3/2)^2 (2-3/2)^2 (3/2-1)^5$$
$$= (+9/4)(1/4)(1/32)$$
$$= (9/16)(1/32) \quad \text{small!}$$

$$f(5/2) = (3-5/2)^2 (2-5/2)^2 (5/2-1)^5$$
$$= (1/4)(1/4)(243/32)$$
$$= (1/16)(243/32)$$

larger than f(3/2) but still small

Leading coeff is Positive.



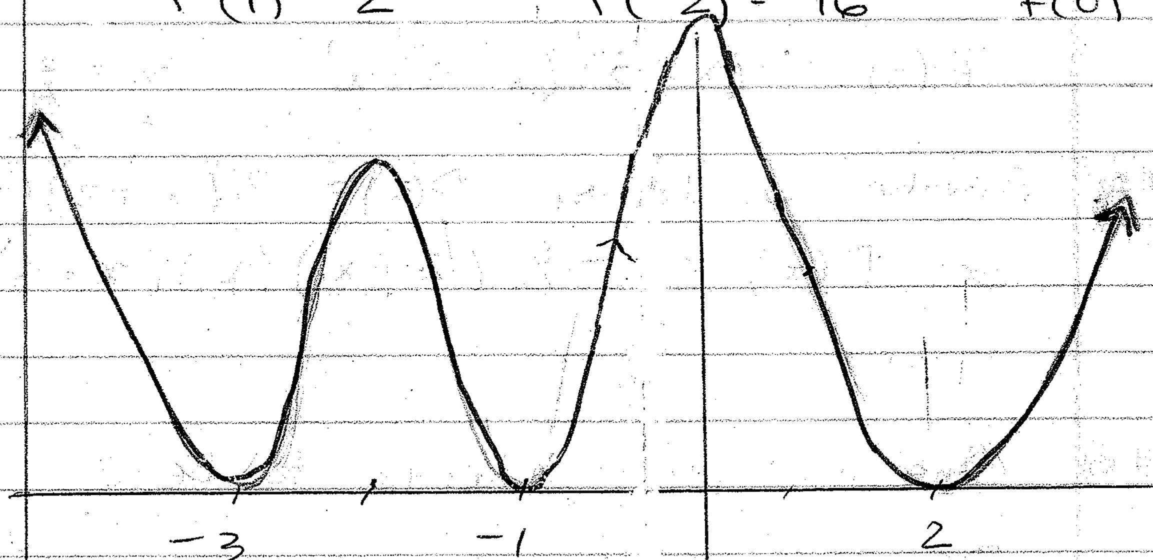
#3f y-int: 36

roots: 2, −3, −1   M even for all

lead coeff: positive   ↖ ⁓ ↗

We can't show y-int accurately b/c it's so high, but we ought to inspect between the roots:

$f(1) = 2$ $f(-2) = 16$ $f(0) = 36$



#4) Construct the polynomial from the model

$$P(x) = (x-r_1)^{m_1}(x-r_2)^{m_2}\cdots(x-r_k)^{m_3}$$

where r_i are roots + m are multiplicities. Note, the sum of the multiplicities is the degree n.

$$P(x) = (x+2)^1(x-5)^1x^1 \qquad n=3$$

#5 Another would be $P(x) = 3(x+2)(x-5)\cdot$ or $P(x) = -\frac{1}{2}(2+x)(x)(x-5)$

#6) One multiplicity needs to be 2.

$$P(x) = x(x+2)^2(x-5)$$

#7) $P(x) = x^2(x+2)(x-5)^4$

#8) Rational root property - consider all factors of a_0 + a_n as fractions: $a_nx^n + \cdots + a_0 = P(x)$

Possible roots : $\{p/q \mid p \text{ divides } a_0,\ q \text{ divides } a_n\}$

Sec 5.1 con'd

#9d) $f(x) = -x^4 - 1$

$= -(x^4+1)$ This has no real roots!

Why not? Because

$x^4 + 1 = 0$

cannot be solved with real numbers.

i.e., $x^4 \neq -1$ if x must be real.

#9e) $f(x) = 2x^2 - 17x - 55$

$= (2x+5)(x-11) = 0$

$x = -5/2, 11$

We didn't need the rational root theorem, so here's a guess at how it will show up on a test:

Given the polynomial

$f(x) = x^5 - 2x^4 + 3x^2 - 6$

is it possible that $-1/2$ is a root? Why or why not?

Answer No, b/c $-1/2$ is not made up of possible factors of 6 divided by factors of 1.

Sec. 5.2 HW Solutions

#3 The equation of ~~a~~ the horizontal line through a given point (a, b) is $\boxed{y = b.}$

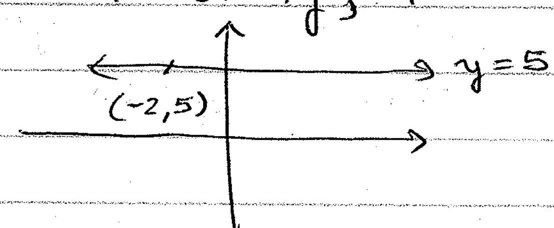
This is reliable – no need to look at slope-intercept form $(y = mx + b)$ but if you do, you'll notice that because $m = 0$ for a horizontal line, $y = mx + b$ becomes $y = 0 \cdot x + b$, or

$$y = b$$

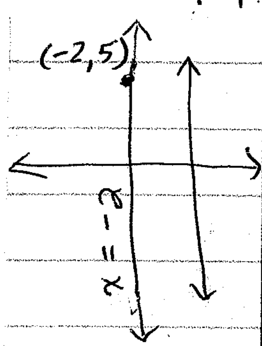
This is called the constant function. It says y is equal to b for all values of x. Thus, the equation of the horizontal line through $(-2, 5)$ is

$$\boxed{y = 5}$$

Slope m is zero and the y-intercept is the constant value of y, in this case, 5.



#4 The equation of the vertical line through a given pt. (a, b) is $\boxed{x = a}$. We don't even try to get this from slope-intercept form, since a vertical line has undefined slope. A vertical line is not even a fcn. It is a "relation" that says $x = a$ for all y. Through $(-2, 5)$, $\boxed{x = -2}$, slope is undefined, and there is no y-intercept



Sec 5.2 Con'd

#5) Since slope of a line between any two pts is $\frac{\Delta y}{\Delta x}$, it should not matter the order in which the difference between the chosen pts. is taken. Thus, between (x_1, y_1) & (x_2, y_2):

$$\frac{y_1 - y_2}{x_1 - x_2} = \frac{y_2 - y_1}{x_2 - x_1}$$

To show this algebraically, we can work on only one side, showing it's equal to the other.

$$\frac{y_1 - y_2}{x_1 - x_2} = \frac{-(y_2 - y_1)}{-(x_2 - x_1)} = \frac{y_2 - y_1}{x_2 - x_1}$$

#6 From pt $(2,3)$ we can arrive at another pt via knowledge of slope. Given $m = -2$, or $\frac{-2}{1}$, go 2 units down and 1 unit right, arriving at $(3,1)$ or 2 units up and 1 unit left, arriving at $(1,5)$.

#7. To go from $(-1,-4)$ to a pt. in the first quadrant, you must go at least far enough right and up to make both negative values positive. The slope is 1, that is, $1/1$, so for every unit up (or down) you must go 1 unit right (or left). Choosing 5 spaces up and 5 right works to get to quad one.

$$(-1,-4) \longrightarrow (4,1)$$

#8 Use $m = \frac{\Delta y}{\Delta x}$. It does not matter which pt you choose for (x_1, y_1) or which for (x_2, y_2). Reduce answer. For which pair is m undefined (Answer: The pts having same x value.)

#9 a) $y - 1 = 3(x+2) \rightarrow \boxed{y = 3x + 7}$ $m = 3$, $y\text{-int} = 7$, $x\text{-int} = -7/3$

b) $f(x) = 2x + 3 \rightarrow \boxed{y = 2x + 3}$ $m = 2$, $y\text{-int} = 3$, $x\text{-int} = -3/$

c) $g(x) = -x + 5 \rightarrow \boxed{y = -x + 5}$ $m = -1$, $x\text{-int} = 5$, $y\text{-int} = 5$

d) $\boxed{y = -\frac{2}{3}x + \frac{1}{2}}$, $m = -2/3$, $y\text{-int} = 1/2$, $x\text{-int} = 3/4$

e) $\boxed{y = 7}$, $m = 0$, $y\text{-int} = 7$, no x-int.

f) $\boxed{x = -2}$ m undefined, no y-int. $x\text{-int} = -2$

#10 a) $\boxed{y = \frac{2}{3}x + 2}$ (Note that the y-int is given in the pt $(0, 2)$)

b) $y - y_1 = m(x - x_1)$ is the best equation to use when given a pt other than the y-int and the slope. Thus, $y - {}^{-}3 = -1(x - 3)$.

Simplify: $y = -x + 3 - 3 = -x$ $\boxed{y = -x}$

c) $y - y_1 = m(x - x_1)$

$y - 2 = 5(x - {}^{-}1)$ $\boxed{y = 5x + 7}$

Sec 5.2 - con'd

#13) If $f^{-1}(5)=0$ then $f(0)=5$

If $f^{-1}(0)=-3$ then $f(-3)=0$

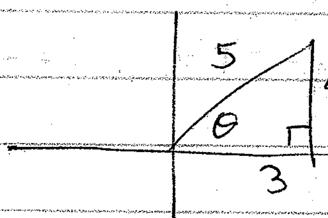
~~Between these two pts, the slope of f is~~

$$m = \frac{0-5}{-3-0} = \frac{5}{3}$$

We are given b (the y-int is 5)

So $\boxed{y = \frac{5}{3}x + 5}$

#14



Since $\cos\theta = \frac{adj}{hyp} = \frac{3}{5}$

the other leg is 4 $(3^2+4^2=5^2)$.

Hence, $m = \frac{4-0}{3-0} = \boxed{\frac{4}{3}}$

#15 ~~$f(x)=mx+b$~~ $\longrightarrow$ $y=mx+b$

$x = my+b \longrightarrow \frac{x-b}{m} = y = f^{-1}(x)$

$\therefore \boxed{f^{-1}(x) = \frac{1}{m}x - \frac{b}{m}}$

$m \neq 0$ for f since the slope of f^{-1} is $1/m$, which is undefined when $m=0$.

#16 ~~Parallel lines have the same slope~~

~~Perpendicular lines have negative reciprocal slopes.~~

$3y = x-15$ becomes $y = \frac{1}{3}x - 5$ (line 1)

$y+2 = -3x$ becomes $y = -3x-2$ (line 2)

Clearly $m_{line\,1} = \frac{-1}{m_{line\,2}}$

So the lines are $\perp$

#17) $2x + y = -4$ given, so $m = -2$

a) For a parallel line, $m = -2$ also.

Given pt. $(2, -1)$, $y - {}^-1 = -2(x-2)$

or $\boxed{y = -2x + 3}$

b) For a perpendicular line, $m = 1/2$.

Given pt $(2, -1)$, $y - {}^-1 = \frac{1}{2}(x-2)$

or $\boxed{y = \frac{1}{2}x - 2}$

#18) For dist + midpt, just be careful of your computation. Don't mix up x + y. And leave answer in reduced fraction form, not decimal.

Also, if an answer is a radical, simplify it as far as possible.

Ex Dist between $(0, -2)$ + $(6, 0)$

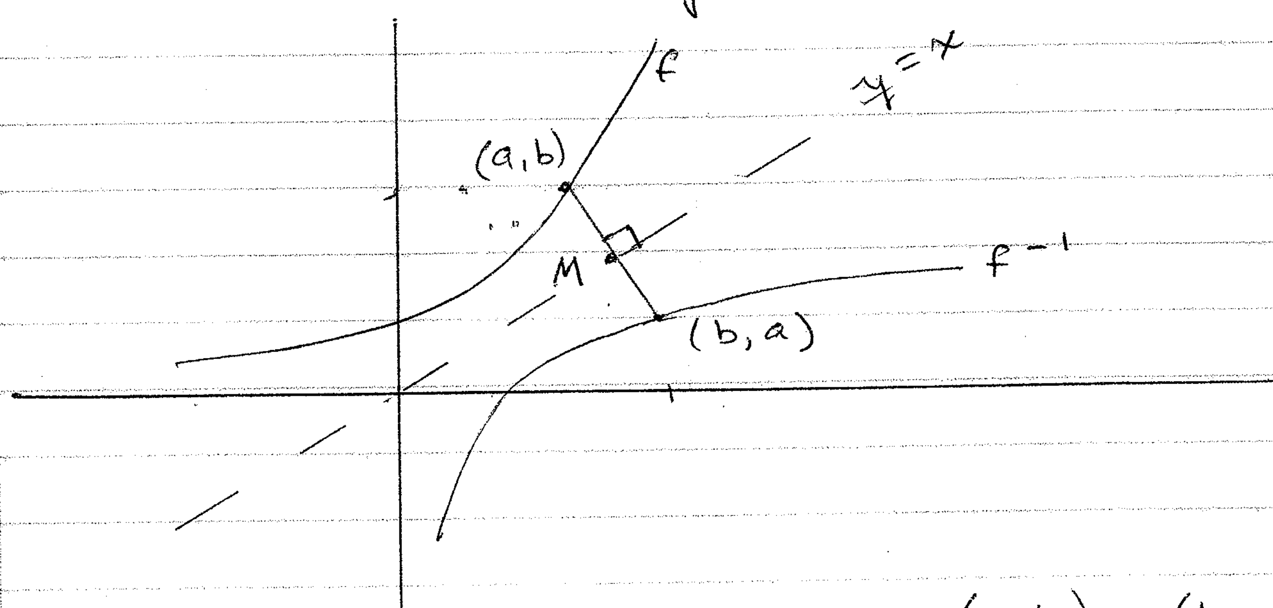
$$d = \sqrt{(0-6)^2 + (-2-0)^2}$$

$$= \sqrt{36 + 4}$$

$$= \sqrt{40} = 2\sqrt{10}$$

#19) We're given that f and f^{-1} are inverses and asked to show that the segment between any 2 pts (a, b) and (b, a) has a slope of -1 and that $y = x$ bisects the segment at right angles (i.e., it's the $\perp$ bisector).

If both of these obtain, then we've shown the symmetry of f with f^{-1} across the line $y = x$.



The slope of segment connecting (a, b) + (b, a) is $\dfrac{b-a}{a-b} = -1$. Good.

If $y = x$ is the $\perp$ bisector of the segment then M is the midpt, first of all. Halfway between (a, b) + (b, a) is the pt $\left(\dfrac{a+b}{2}, \dfrac{b+a}{2}\right)$. Is this pt. on the segment at $y = x$? Of course, since $\frac{a+b}{2} = \frac{b+a}{2}$.

Since the slope of the segment is the negative reciprocal of $y = x$ as we have shown, we are done!