Sec 5.1 HW Solutions

#1. $P(x) = a_n x^n + a_{n-1} x^{n-1} + \cdots + a_2 x^2 + a_1 x + a_0$

where the a_i are coefficients

+ the n are non-negative integers.

So ... only a, c, e, g are polynomials.

note e) 4 is the polynomial with all coeffs $= 0$ except a_0, which is 4.

$P(x) = 4$ is a horizontal line through $y = 4$.

#2. The y-intercept is always $P(0)$.

$P(0) = a_0$ but you don't need to multiply out the terms to compute this. Just substitute $x = 0$ into the given equation.

a) $f(0) = (-2)^3(-1) = 8$

b) $f(0) = 0(0+1)^4(0+3) = 0$

c) $f(0) = -(0+1)(0-2)(0+3) = 6$

d) $f(0) = (1)(4)(0) = 0$

e) $f(0) = (3)^2(2)^2(-1)^5 = -36$

f) $f(0) = 2^2 \cdot 3^2 \cdot 1^4 = 36$

The leading coeff is a_n, but you don't need to multiply out the terms to find it. Just multiply the coeffs of factored x terms.

However, if a factor is raised to a power other than 1, adjust the coeff. accordingly:

a) $(x-2)^3(x-1)^1 \longrightarrow 1^3 \cdot 1^1 = 1$

b) $x(x+1)^4(x+3)^1 \longrightarrow 1 \cdot 1^4 \cdot 1 = 1$

c) $-(x+1)(x-2)(x+3) \longrightarrow -1 \cdot 1 \cdot 1 = -1$

d) $(2x+1)(-x+4)x^2 \longrightarrow 2 \cdot -1 \cdot 1 = -2$

e) $(3-x)^2(2-x)^2(x-1)^5 \rightarrow (-1)^2 \cdot (-1)^2 \cdot 1^5$
$= 1$

f) $(2-x)^2(3+x)^2(x+1)^4 = (-1)^2 \cdot 1^2 \cdot 1^4$
$= 1$

The ~~deg~~ degree is the sum of the exponents on the factors — again, no need to multiply out:

a) $n = 3+1 = 4$
b) $n = 1+4+1 = 6$
c) $n = 2+2+5 = 9$
d) $n = 1+1+2 = 4$
e) $n = 2+2+5 = 9$
f) $n = 2 + 2+4 = 8$

Roots (values of x where $f(x)=0$; also called the x-intercepts or the "zeroes")

a) $f(x) = (x-2)^3 (x-1) = 0$

when $(x-2)^3 = 0$ or $x-1=0$

so $x-2=0$ $\quad$ $x-1=0$

$x=2$ $\quad$ $x=1$

"Multiplicity" refers to number of times a factor $x-r$ occurs in the factorization of a polynomial. The multiplicity (call it M) of the factor $(x-r)$ is also the mult of the root r itself.

So M of $r_1 = 2$ is 3

M of $r_2 = 1$ is 1.

Comment: A template for a fully factored polynomial is given as

$$(x-r_1)(x-r_2) \cdots (x-r_n)$$

where $r_1, r_2, \ldots, r_n$ are the roots. The minus sign is the convention; the signs of the individual roots may be (+) or (−). e.g.:

$(x+2)(x+7)(x-1)$ has roots

$r_1 = -2$, $r_2 = -7$, $r_3 = 1$

b) $f(x) = x^1(x+1)^4(x+3)^1$

$r_1 = 0, \quad M_1 = 1$

$r_2 = -1, \quad M_2 = 4$

$r_3 = -3, \quad M_3 = 1$

c) $f(x) = -(x+1)^1(x-2)^1(x+3)^1$

$r_1 = -1, \quad M_1 = 1$

$r_2 = 2, \quad M_2 = 1$

$r_3 = -3, \quad M_3 = 1$

d) $f(x) = (2x+1)^1(-x+4)^1 x^2$

$r_1 = -\frac{1}{2}, \quad M_1 = 1$

$r_2 = 4, \quad M_2 = 1$

$r_3 = 0, \quad M_3 = 2$

e) $f(x) = (3-x)^2(2-x)^2(x-1)^5$

$r_1 = 3, \quad M_1 = 2$

$r_2 = 2, \quad M_2 = 2$

$r_3 = 1, \quad M_3 = 5$

f) $f(x) = (2-x)^2(3+x)^2(x+1)^4$

$r_1 = 2, \quad M_1 = 2$

$r_2 = -3, \quad M_2 = 2$

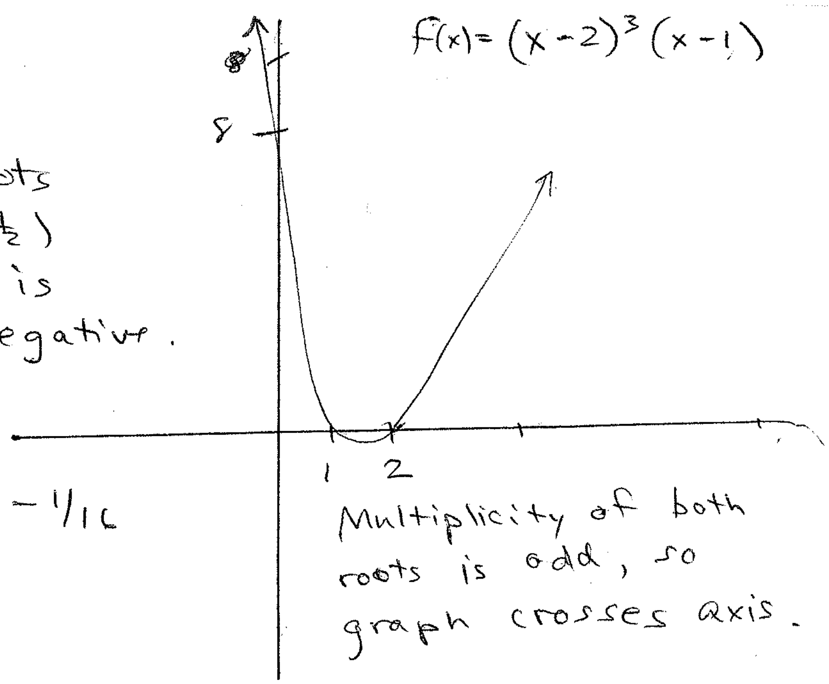
$r_3 = -1, \quad M_3 = 4$

Sketches: Besides the info above, look at shape of mother fcn, and sign of lead coeff. Also, if M is odd, the graph crosses the x-axis, and if M is even, it only touches it.

It's a good idea to plot a pt. or two between roots.

a)

$f(x) = (x-2)^3(x-1)$



Between roots (say $x = 1\frac{1}{2}$) the graph is slightly negative.

$f(3/2) = -1/16$

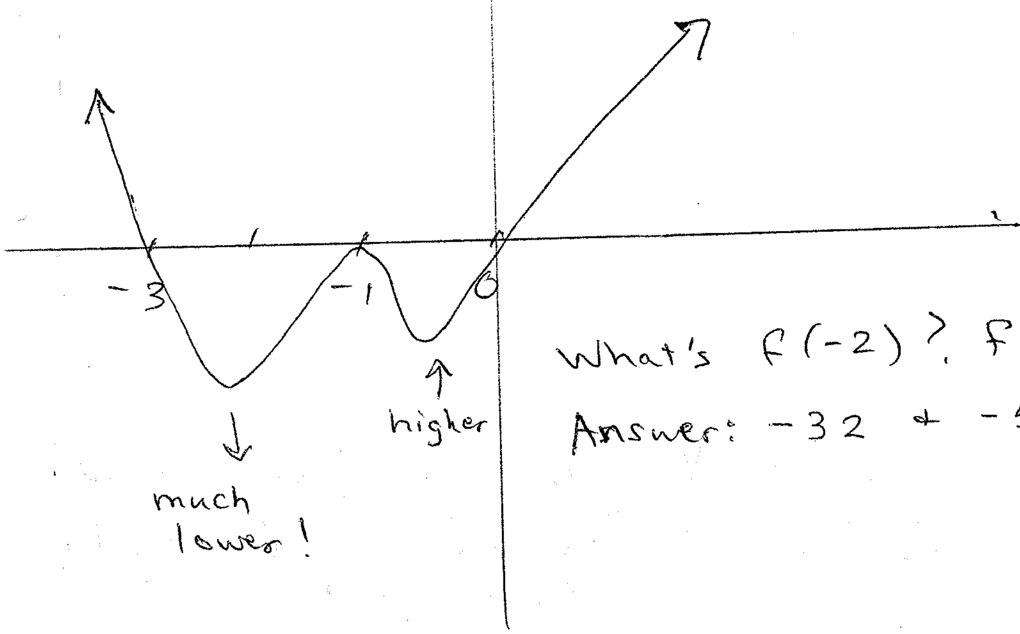
Multiplicity of both roots is odd, so graph crosses axis.

b) y-int = 0, roots: -3, -1, 0

M of -3 + 0 is one. (Crosses)

M of -1 is four. (Touches)

$y = x(x+1)^4(x+3)$

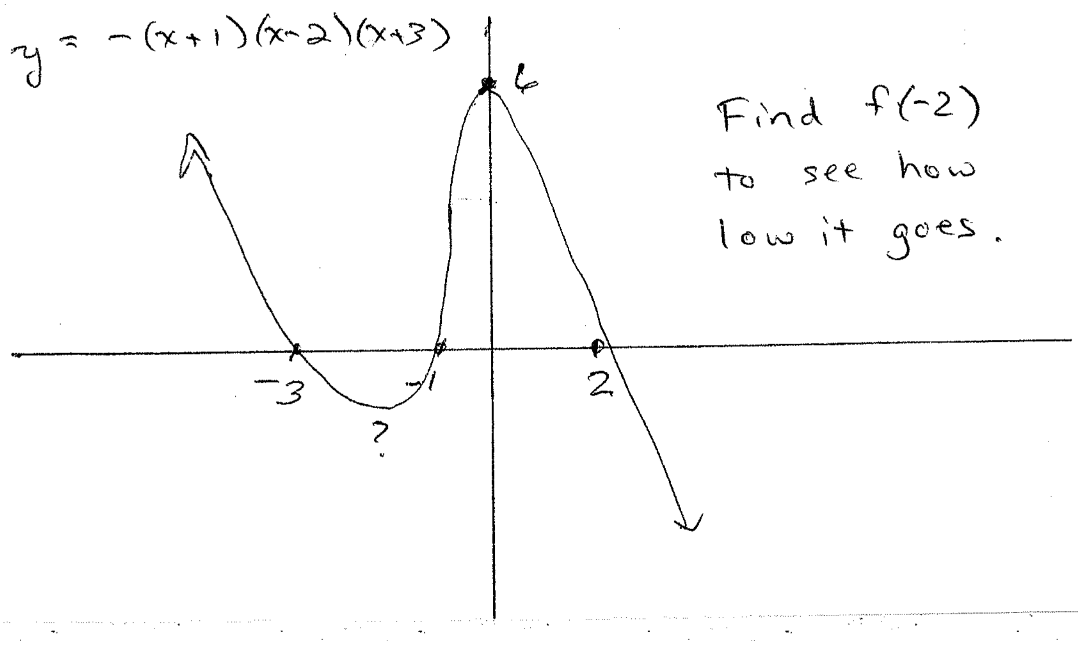


What's $f(-2)$? $f(-\frac{1}{2})$?

Answer: -32 + -5/64

c) y-int: 6 roots: -1, 2, -3 M=1 for all (crosses at all roots)

$y = -(x+1)(x-2)(x+3)$



Find $f(-2)$ to see how low it goes.