Ch. 7.2 HW Solutions

#1) We know the triangle inequality holds

that $|a+b| \leq |a| + |b|$

but for certain a, b, $|a+b| \neq |a| + |b|$

They are any a, b with opposite signs.

Try: $a = -2$, $b = 7$:

$$|-2+7| = |5| = 5 \neq |-2| + |7| = 2+7 = 9$$

Done

exception ~~a = B~~ $a = -b$

For any same signed a, b, we see equality.

$|a| + |-b| = |a| + |a| = |2a|$

Try: $a = 2$, $b = 7$

$$|2+7| = 9 = |2| + |7|$$

Try $a = -2$, $b = -7$

$$|-2 + -7| = |-9| = 9 = |-2| + |7|$$

#2) We know the "other side" of the triangle inequality holds that

$$|a| - |b| \leq |a-b|$$

but for certain a, b, $|a| - |b| \neq |a-b|$

~~For any~~ $a < b$ or opp signs

Again, this is true for ~~same~~ opposite-signed a, b.

Try $a = 2$, $b = -7$

$$|2| - |-7| = 2 - 7 = -5 \neq |2 - -7| = 9$$

Check for same-signed a, b to see equality.

#3) What values x give $|3x-4| = 3x-4$?

By definition:

$$|3x-4| = \begin{cases} 3x-4, & 3x-4 \geq 0 \\ & x \geq 4/3 \\ -(3x-4), & 3x-4 < 0 \\ & x < 4/3 \end{cases}$$

Solving $3x-4 \geq 0$ gives $x \geq 4/3$ //

~~Solving $(3x-4)$~~

Solving $3x-4 < 0$ gives $x < 4/3$ //

(geometrically)

#4 $|x-5| \neq x+5$ since the distance between x & 5, which is denoted by $|x-5|$ is by definition (algebraic)

$$|x-5| = \begin{cases} x-5, & x \geq 5 \\ -x+5, & x < 5 \end{cases}$$

~~Neither~~ Neither of the fcns. is $x+5$.

Also, $x+5$ can be negative, which $|x-5|$ will never be.

#5. a) $|2x| = x+1$ when $2x = x+1$ ①

$|x| = c$ that is, $\boxed{x=1}$ //

means OR when $2x = -(x+1)$

$X = C$ that is, $2x = -x-1$

or $3x = -1$ ②

$X = -C$ $\boxed{x = -1/3}$ //

#5b) $|-3x+6| = 9x$

① $-3x+6 = 9x$

$-12x = -6$

$\boxed{x = 1/2}$

<u>Chk.</u> $|-3 \cdot \frac{1}{2} + 6|$

$= |-\frac{3}{2} + 6| = 4\frac{1}{2}$

$9x = 9(\frac{1}{2}) = \frac{9}{2}$

$4\,1/2 = 9/2$

② $-3x+6 = -9x$

$6x = -6$

$\boxed{x = -1}$ X

<u>Chk</u> $|-3(-1)+6| = 9$

$9x = 9(-1) = -9$

discard $x = -1$ //

$x = 1/2$

#5c) $|2x-5| = 9$ Either $2x-5 = 9$

or $2x-5 = -9$

① $2x-5=9$

$\boxed{x=7}$

② $2x-5 = -9$

$\boxed{x = -2}$ Both check //

#5d) $x - |x| = 1 \xrightarrow{\text{tactic}} |x| = x-1 \longrightarrow$ ① $x = -x+1$

or

② $x = x-1$

① $x = -x+1 \rightarrow 2x = 1 \rightarrow x = 1/2$

<u>Chk</u> $1/2 - |1/2| = 0 \neq 1$ Discard $x = 1/2$

② $x = x-1 \rightarrow 0 = -1$ a contradiction. ~~Discard~~

So there's no solution. //

#5e) $|x-10| = x^2 - 10x$

① $x - 10 = x^2 - 10x$ ② $x - 10 = 10x - x^2$

$x^2 - 11x + 10 = 0$ $x^2 - 9x - 10 = 0$

$(x-10)(x-1) = 0$ $(x-10)(x+1) = 0$

$x = 10, 1$ or $x = 10, -1$

Checking:

① $x = 10$: $|x-10| = |10-10| = 0 \stackrel{?}{=} x^2 - 10x = 100 - 100$ ✓

$x = 1$: $|x-10| = |1-10| = 9 \stackrel{?}{=} x^2 - 10x = 1 - 10 = -9$ ✗

So far, $x = 10$ is our soln. Check ②

② $x = 10$: Already checked.

$x = -1$: $|x-10| = |-1-10| = 11 \stackrel{?}{=} (-1)^2 - 10(-1) = 11$ ✓

$$\boxed{x = -1 \text{ or } 10}$$

#5f) $\left|\dfrac{x+3}{2x-1}\right| = 2$ Solve for both ± 2.

$\left|\dfrac{x+3}{2x-1}\right| = 2$

You can "cross-multiply" here, since it's not an inequality, so there's no danger of losing half the soln.

$x + 3 = 2(2x-1)$

$\boxed{x = 5/3}$

(checks)

(2) ~~2x~~ $\dfrac{x+3}{2x-1} = -2$ (This is the same as saying $-\left(\dfrac{x+3}{2x-1}\right) = 2$, per the algebraic definition)

OK to multiply across:

$x + 3 = -4x + 2$

$\boxed{x = 1/5}$ Also checks. //

#5g) $|x^2 + x| = |x - 15|$ (See Ex. 7.2.7 for a similar problem.)

$\boxed{\text{Method 1}}$

The four scenarios for solving equations of the form $|f(x)| = |g(x)|$ are

(1) $f(x) = g(x)$

(2) $f(x) = -g(x)$

(3) $-f(x) = g(x)$

(4) $-f(x) = -g(x)$

There's really just two scenarios: (1) + (2)

~~Why not just give as the rule for this the one we have for $|f(x)| = c$?~~

~~We could, though geometrically it might look different~~

$\boxed{\text{Method 2}}$ Use the fact that $|a|^2 = a^2$ (p. 202 props) ~~to~~ eliminate $|\;|$ & solve like the usual 2nd degree equations.

Both Method 1 + 2 require we check our possible answers.

$\boxed{\text{Method 1}}$ Solving ① $x^2 + x = x - 15$

$\rightarrow \quad x^2 + 15 = 0$ no soln

② $x^2 + x = -(x - 15)$

$x^2 + 2x - 15 = 0$

$(x-3)(x+5) = 0$

$\boxed{x = 3, -5}$ These must be checked.

$x = 3$: $|x^2 + x| = |9 + 3| = 12$

$|x - 15| = |3 - 15| = 12$ ✓

$x = -5$: $|x^2 + x| = |20| = 20$

$|x - 15| = |-5 - 15| = |-20| = 20$ ✓

~~#5g)~~

~~#5h) $|1 - 2x| = 3 + |x + 5|$~~ ~~(See Ex. 7.2.8 for a similar problem)~~

$\boxed{\text{Method 2}}$ $|x^2 + x|^2 = |x - 15|^2$

$(x^2 + x)^2 = (x - 15)^2$

$x^4 + 2x^3 + \cancel{x^2} - \cancel{x^2} + 30x - 225 = 0$

$x^4 + 2x^3 + 30x - 225 = 0$

Apparently, this factors as $(x-3)(x+5)P(x)$ where $P(x)$ is found by long division of $(x-3)(x+5)$ into $x^4 + 2x^3 + 30x - 225$. I didn't come up with this by testing possible roots via the rational root thm.

but by substituting $x = -5$ + $x = 3$ from Method 1 to see if these values worked, I imagine we'll stick with Method 1 for this kind of problem.

#5h) $|1-2x| = 3 + |x+5|$

This is like Ex. 7.2.8 It requires that four possible scenarios be considered. We'll see that inconsistencies can arise between a possible solution and the condition imposed on the domain for each possible solution.

(1) If $1-2x \geq 0$ and $x+5 \geq 0$

then we'd have the original equation with abs. value removed:

Solve $1 - 2x = 3 + x + 5 \longrightarrow x = -7/3$ Valid?

To see if this is valid, we'd look at the domain and whether $x = -8/3$ satisfies it. The domain is the "If $1-2x \geq 0$ and $x+5 \geq 0$" condition. That is $x \leq 1/2 \cap x \geq -5$, $[-5, 1/2]$

~~$x = -7/3$ is in the domain~~

$x = -7/3$ is in $[-5, 1/2]$. It also checks in the original problem: $|1-2x| = 3 + |x+5|$

② If
$1-2x \geq 0$ and $x+5<0$

then we solve $1-2x = 3-(x+5)$

$\rightarrow 1-2x = 3-x-5 \rightarrow \boxed{x=3}$ Valid?

The domain is given in the "If".

$1-2x \geq 0 \rightarrow x \leq 1/2$ } Intersect at

$x+5<0 \rightarrow x<-5$ } $(-\infty, -5)$

$x=3$ is not on this domain, so discard.

③ If $1-2x<0$ and $x+5<0$

then we solve $-(1-2x) = 3-(x+5)$

$\rightarrow -1+2x = 3-x-5 \rightarrow \boxed{x=-1/3}$

Valid?

The domain from the "If"

is found:

$1-2x<0 \rightarrow x>1/2$ } Intersection

$x+5<0 \rightarrow x<-5$ } is ϕ

So discard the soln. (In fact, you do not even have to solve on domains of the empty set!)

④ Finally! If $1-2x<0$ and $x+5 \geq 0$

we'd solve $-1+2x = 3+x+5$

$\rightarrow \boxed{x=9}$ Valid?

The domain is $x>1/2$ and $x \geq -5$,

that is $(1/2, \infty)$. $x=9$ is in here.

And it checks in $|1-2x| = 3+|x+5|$ //

#5i) $|x|^2 + |x| - 12 = 0$

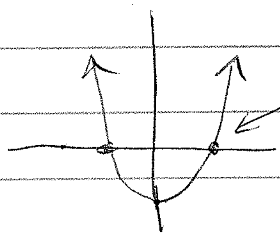
Noting that $|x|^2 = x^2$, we simplify:

$x^2 + |x| - 12 = 0 \longrightarrow x^2 - 12 = -|x|$

which is solved the usual way: $\boxed{|x| = 12 - x^2}$

① $x = x^2 - 12$ if $x^2 - 12 \geq 0$.

What does $x^2 - 12 \geq 0$ mean? It's where is the parabola $y = x^2 - 12 \geq 0$, which is on $(-\infty, -2\sqrt{3})$ or $(2\sqrt{3}, \infty)$.

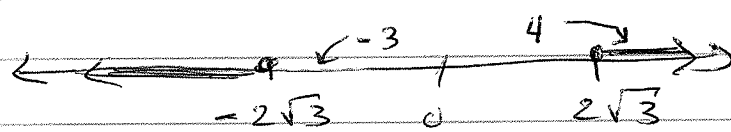


roots are $x = \pm\sqrt{12} = \pm 2\sqrt{3}$.

Solving $x = x^2 - 12 \rightarrow x^2 - x - 12 = 0$

$\rightarrow (x-4)(x+3) = 0 \rightarrow \boxed{x = 4 \text{ or } -3}$

Are either of these in the domain?



Only $x = 4$ is valid. Checking: $|4|^2 + |4| - 12 \neq 0$

② Solving $x = 12 - x^2$ if $x^2 - 12 < 0$

gives $x = 3$ or -4 on $(-2\sqrt{3}, 2\sqrt{3})$

Only $x = 3$ is valid here.

∴ The solns. are 3 & -3

#6) (a) $\dfrac{|x+2|}{x+2}$ for $x \neq -2$

Either $\dfrac{x+2}{x+2} = 1$ if $x > -2$

or $\dfrac{-x-2}{x+2} = -1$ if $x < -2$

(b) $\left|\dfrac{x^2-4}{3x+6}\right|$ for $x \neq -2$

$$\frac{x^2-4}{3x+6} = \frac{\cancel{(x+2)}(x-2)}{3\cancel{(x+2)}} = \frac{x-2}{3}.$$

if $x^2-4 \geq 0$ and $3x+6 \geq 0$

that is $x^2 \geq 4$ $\quad x \geq -2$

$(-\infty, -2) \cup (2, \infty) \cap (-2, \infty)$

which is $(2, \infty)$

(b) $\dfrac{|x^2-4|}{|3x+6|} = \dfrac{\cancel{|x+2|}\,|x-2|}{3\cancel{|x+2|}}$ p. 202

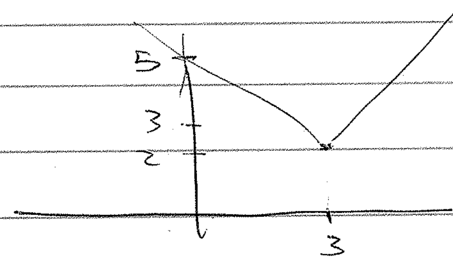
$= \dfrac{|x-2|}{3}$, for $x \neq -2$

$$= \begin{cases} \dfrac{x-2}{3}, & x \geq 2 \\ \dfrac{2-x}{3}, & x < 2 \end{cases}$$

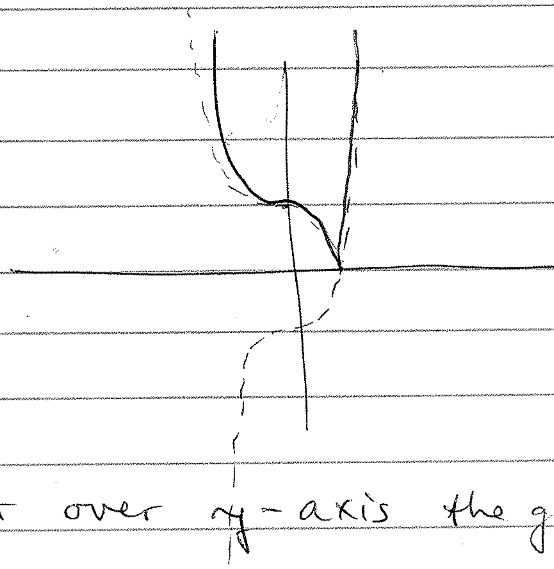
#7) a) $|x-3|=12$ b) $|x-7|=4$

c) $|2x+4|=1$ d) $|x|=6$

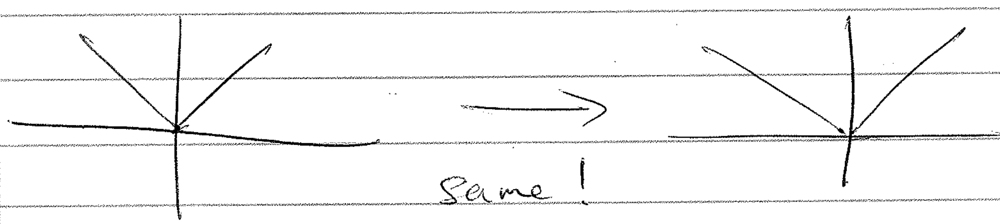
#8) $y=|x-3|+2$



$y=|x^3-1|$



$y=|-x|$ reflect over y-axis the graph $y=|x|$



Same !

Sec 7.3

#1 a) $|x-4| < 6$ b) $|y+1| \geq 8$

c) $|x| \leq 5$ d) $|x| \leq 3$

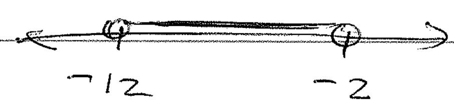
e) $\cancel{|x-1| < 5}$ f) $|x-6| > 4$

$|x-3| < 7$

Counted wrong :(

#2 a) $|x+7| < 5 \rightarrow x+7 < 5 \Rightarrow x < -2$

and $x+7 > -5 \rightarrow x > -12$



b) $|1-2x| > 5 \longrightarrow 1-2x > 5 \Rightarrow x < -2$

or $1-2x < -5 \rightarrow x > 3$

c) $\left|1 - \frac{2x}{3}\right| \leq 1$

$-1 \leq 1 - \frac{2x}{3} \leq 1$

$-3 \leq 3 - 2x \leq 3$

$-6 \leq -2x \leq 0$

$\boxed{3 \geq x \geq 0}$

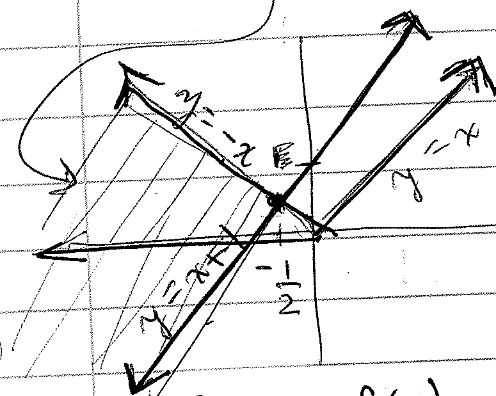
d) $\left|\frac{4-5x}{2}\right| \geq 1 \rightarrow |4-5x| \geq 2$

So $4-5x \geq 2$ or $4-5x \leq -2$

etc.

e) $|\cos x - 243| \leq -2$ No solution, since abs. value is never negative. //

f) $|x| \geq x+1$ Compare graphs of fcns. $y = |x|$ and $y = x+1$.



We see that on $(-\infty, -\frac{1}{2})$ the abs value fcn is above the line $y = x+1$ (ie. where $y = -x$ intersects $y = x+1$)

solve $-x = x+1$

$-2x = 1$

$x = -\frac{1}{2}$

Since $f(x) = |x|$

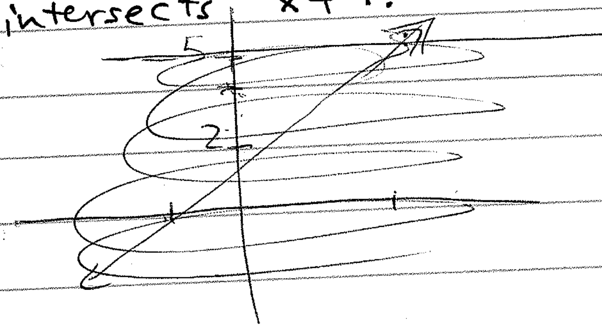
$f(x) = \begin{cases} x, & x \geq 0 \\ -x, & x < 0 \end{cases}$

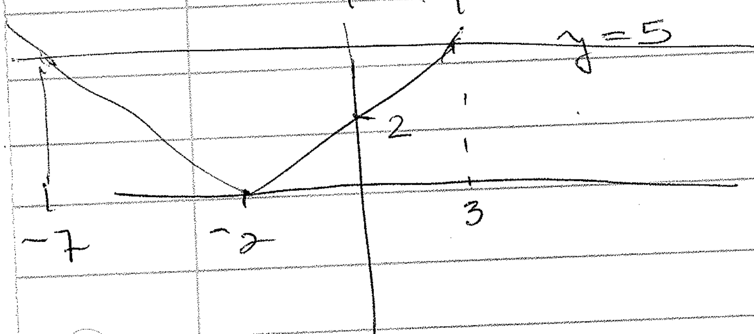
you need to consider the equation where $|x|$ intersects $x+1$.

g) $3|x+2| - 5 > 10$

$3|x+2| > 15$

$|x+2| > 5$





$(-\infty, -7) \cup (3, \infty)$

#2h)

$|x-1| > |3x-5|$

First determine where this is zero.
$3x-5=0 \rightarrow x = 5/3$

Notice that, as long as $x \neq 5/3$, we could divide both sides by $|3x-5|$ and solve $\frac{|x-1|}{|3x-5|} > 1$. We don't (necessarily) discard $x = 5/3$ until we see if it's in the solution: $|5/3 - 1| \overset{?}{>} |3(5/3)-5| = 0$

Of course it is! $2/3 > 0$ ✓

Continue solving $\frac{|x-1|}{|3x-5|} > 1$

Squaring will eliminate $|\ |$.

$$\left|\frac{x-1}{3x-5}\right|^2 = \frac{(x-1)^2}{(3x-5)^2} > 1^2$$

$$\rightarrow x^2 - 2x + 1 > 9x^2 - 30x + 25$$

$$8x^2 - 28x + 24 < 0$$

$$2x^2 - 7x + 6 = 0$$

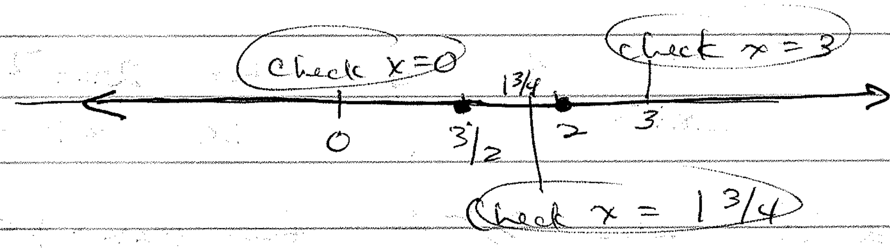
$$(2x - ?)(x + ?) = 0$$

Use the quadratic formula:

$$x = \frac{7 \pm \sqrt{49 - 4 \cdot 2 \cdot (+6)}}{2 \cdot 2} = \frac{7 \pm 1}{4}$$

$x = 2$ or $3/2$

Now check the values of x in the intervals marked off by $x=2$ + $3/2$.



Check these into the original inequality:

$\underline{x=0}$: $|0-1| \overset{?}{>} |0-5| \longrightarrow 1<5$ ✗

$\underline{x=1\,3/4}$: $|1\,3/4-1| \overset{?}{>} |3(1\,3/4)-5|$

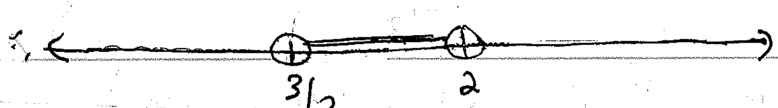
$\longrightarrow 3/4 > 1/4$ ✓

$\underline{x=3}$: $|3-1| \overset{?}{>} |9-5| \longrightarrow 2<4$ ✗

The intervals of valid inequality are

$(-\infty, 3/2) \cup (2, \infty)$

(Notice that $x = 5/3 \in (3/2, 2)$)



#2i) $\dfrac{1}{|2x+7|} \le \dfrac{1}{4}$ (same as $\left|\dfrac{1}{2x+7}\right| \le \dfrac{1}{4}$, which is solved as $|f(x)| \le c$ if $-c \le f(x) \le c$)

So, ① $\dfrac{1}{2x+7} \le \dfrac{1}{4}$ for $2x+7 > 0$ i.e., $x > -7/2$

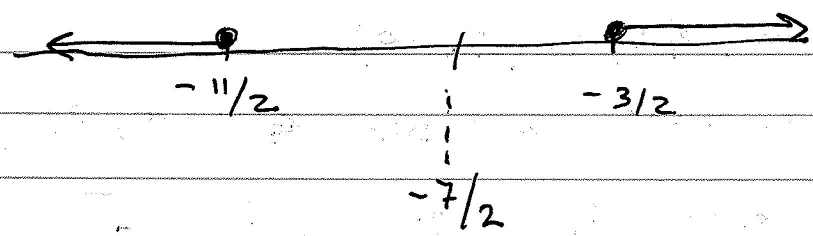
and ② $\dfrac{1}{+2x+7} \ge -\dfrac{1}{4}$ for $+2x+7 < 0$ i.e. $x < -7/2$

① $\dfrac{1}{4} \ge \dfrac{1}{2x+7} \longrightarrow 2x+7 \ge 4$

$\longrightarrow \boxed{x \ge -3/2}$

② $\dfrac{1}{4} \le -\dfrac{1}{2x+7} \longrightarrow 2x+7 \le -4$

$\boxed{x \le -11/2}$



$(-\infty, -11/2) \cup (-3/2 \quad \infty)$