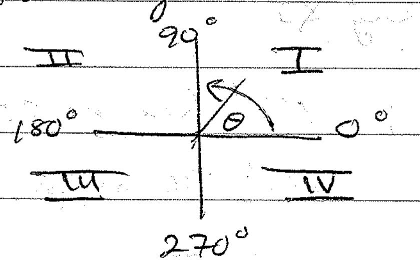
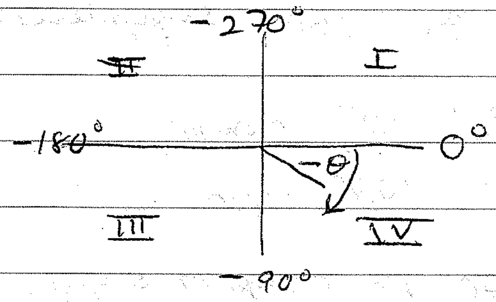
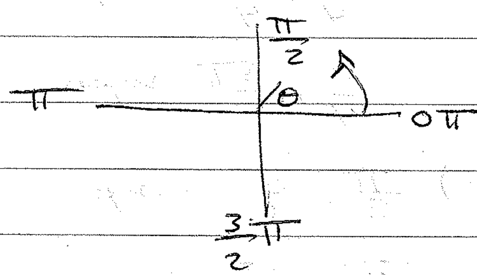
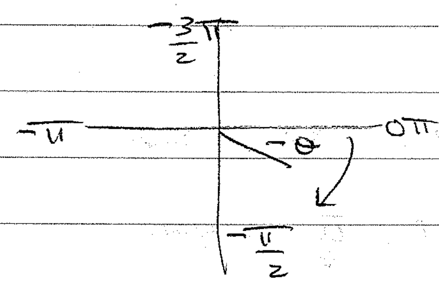
Ch. 8 HW – Trigonometry

#1 a) The quadrant is found by measuring angle θ above the x-axis, counterclockwise (contre-~~mon~~montre in French) for positive θ, below the x-axis (clockwise) for negative θ.

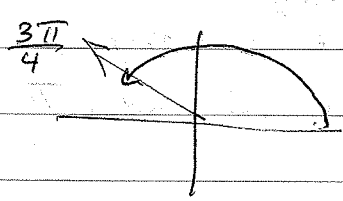






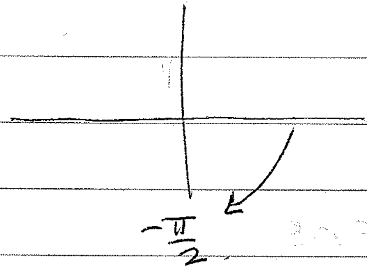


b) Coterminal angle measure is found by adding 2π or subtracting 2π from a given angle θ. You can also add or subtract multiples of 2π to arrive at a coterminal angle.



$$\frac{3\pi}{4} + 2\pi = \frac{11\pi}{4}$$

$$\frac{3\pi}{4} - 2\pi = -\frac{5\pi}{4}$$



$$-\frac{\pi}{2} + 3(2\pi) = \frac{11\pi}{2}$$

$$-\frac{\pi}{2} - 2\pi = -\frac{13\pi}{2}$$

#3) The complement of an angle θ is the angle whose measure sums to 90 $(\pi/2)$ with θ.

The supplement of an angle θ is the angle whose measure sums to 180 (π) with θ.

(a) $30^\circ, 60^\circ$ complementary ∡s

$30^\circ, 150^\circ$ supplementary ∡s

(b) $78^\circ, 12^\circ$ comp.

$78, 102^\circ$ supp.

c) 110° has no complement

$110^\circ, 70$ supp.

d) 270° no comp. or supp.

e) $\frac{\pi}{4}, \frac{\pi}{4}$ comp

$\frac{\pi}{4}, \frac{3\pi}{4}$ supp

f) $\frac{5\pi}{6}$ no comp.

$\frac{5\pi}{6}, \frac{\pi}{6}$ supp

g) $\frac{\pi}{3}, \frac{\pi}{6}$ comp

$\frac{\pi}{3}, \frac{2\pi}{3}$ supp

h) 1 rad — Recall that there are 2π rad in 360°

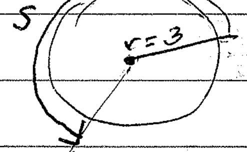
so there are $\frac{2\pi}{4}$ or $\frac{\pi}{2} \approx \frac{3.14}{2}$ in 90°

(Here I'm checking that 1 rad is small enough to have a comp.)

So, 1 rad, $\frac{\pi}{2} - 1$ comp, and 1 rad, $\pi - 1$ are supp.

#5) Length of arc $s = r \cdot \theta$

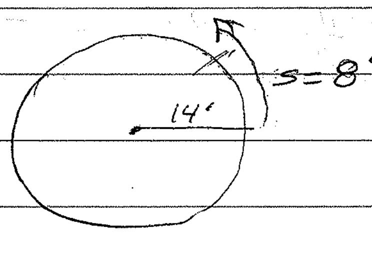
a)



$\theta = 4$ rad

$s = 4 \cdot 3 = 12$ cm

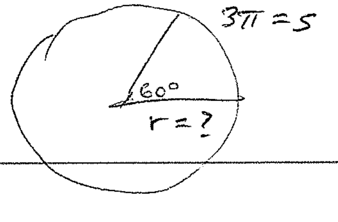
b)



$s = r \cdot \theta$

$\theta = \frac{s}{r} = \frac{8}{14} = \frac{4}{7}$ rad

#5 c)



You must change degree measure to radian measure to apply the formula. $60° = \pi/3$ rad.

$$r = \frac{s}{\theta} = \frac{3\pi \text{ in}}{\pi/3} = 9 \text{ in}$$

#6) ~~Area of a sector~~ $= \frac{1}{2}\theta r^2$ (θ in radian measure)

a) $A = \frac{1}{2} \cdot \boxed{\frac{\pi}{6}} \cdot r^2 = \boxed{\frac{\pi}{12} r^2}$ when $\boxed{\theta = 30°}$

Notice that $30/360 = 1/12$ + Area of sector $= \frac{\text{Area of circle}}{12}$

b) $\theta = 147° = \frac{147}{180}\pi$ rad $= \frac{49}{60}\pi$

$$A = \frac{1}{2}\theta r^2 = \frac{1}{2} \cdot \frac{49}{60} r^2 = \frac{49}{120} r^2$$

Again, notice that $147/360 = 49/120$

c) $\theta = 4$ rad, $A = \frac{1}{2} \cdot 4 \cdot r^2 = 2r^2$

d) We're supposed to prove what we just used! That Area of sector $= \frac{1}{2}\theta r^2$. The ratios seen in (a)-(c) suggest the answer.

$$\frac{\text{sector } \measuredangle}{\text{circle } \measuredangle} = \frac{\theta}{2\pi} \quad + \quad \frac{\text{area sector}}{\text{area circle}} = \frac{A_{sec}}{\pi r^2}$$

~~Setting~~ angle ratios = area ratios in a proportion

gives $\frac{\theta}{2\pi} = \frac{A_{sec}}{\pi r^2}$ or $A_{sec} = \frac{\cancel{\pi} r^2 \theta}{2\cancel{\pi}}$

$$= \frac{r^2\theta}{2}$$