

Add to the formulas from yesterday the following:

• Power reduction formulas: (p. 275)

$$\sin^2 x = \frac{1 - \cos 2x}{2}$$

$$\left(\tan^2 x = \frac{\sin^2 x}{\cos^2 x} \right)$$

$$\cos^2 x = \frac{1 + \cos 2x}{2}$$

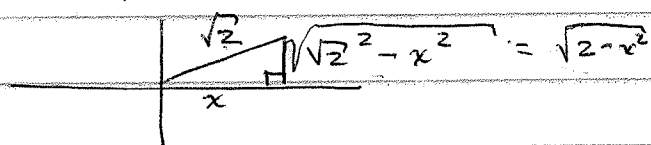
• Half angle formulas (p. 276)

$$\sin \left(\frac{x}{2} \right) = \pm \sqrt{\frac{1 - \cos x}{2}}$$

$$\cos \left(\frac{x}{2} \right) = \pm \sqrt{\frac{1 + \cos x}{2}}$$

Ex 8.8 con'd

#9 Given $\cos \theta = \frac{x}{\sqrt{2}}$ express $\sin(2\theta)$ & $\cos(2\theta)$ in terms of $\cos \theta$.



$$\begin{aligned} \sin 2\theta &= 2 \sin \theta \cos \theta \\ &= \frac{2 \sqrt{2-x^2}}{\sqrt{2}} \cdot \frac{x}{\sqrt{2}} \\ &= \frac{2x \sqrt{2-x^2}}{2} \\ &= x \sqrt{2-x^2} \end{aligned}$$

$$\begin{aligned} \cos 2\theta &= \cos^2 \theta - \sin^2 \theta \\ &= \left(\frac{x}{\sqrt{2}} \right)^2 - \left(\frac{\sqrt{2-x^2}}{\sqrt{2}} \right)^2 \\ &= \frac{x^2}{2} - \frac{2-x^2}{2} \\ &= \frac{2x^2 - 2}{2} = x^2 - 1 \end{aligned}$$

#10 Express $\sinh(2 \sinh^{-1} x)$ as a fn of x with no trig notation:

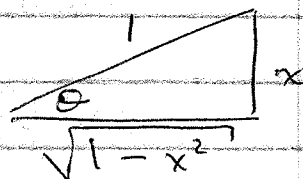
Hint: Use double angle formula

Noticing that $\sinh 2\theta = 2 \sinh \theta \cosh \theta$, we might find this helpful.

$$\sinh(\overbrace{2 \sinh^{-1} x}^{\alpha})$$

First $\longrightarrow$

$$\sinh(2 \sinh \theta)$$



$$\text{so } \cosh \theta = \frac{\sqrt{1-x^2}}{1}$$

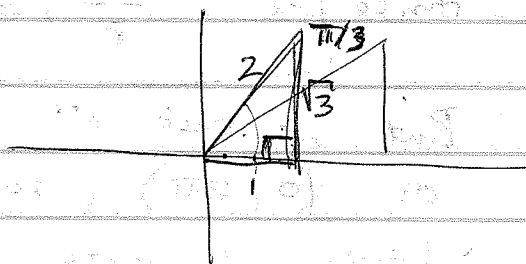
$$\sinh \theta = x$$

Sec 8.9 Solving Trig Equations

Find all solns on $[0, 2\pi)$

#1 a) $\sqrt{3} \csc x - 2 = 0$

$$\csc x = \frac{2}{\sqrt{3}} \rightarrow \csc^{-1}\left(\frac{2}{\sqrt{3}}\right) = \frac{\pi}{3}$$



$$\csc x = \frac{1}{\sin x} = \frac{2}{\sqrt{3}}$$

so find x where

$$\sin x = \sqrt{3}/2$$

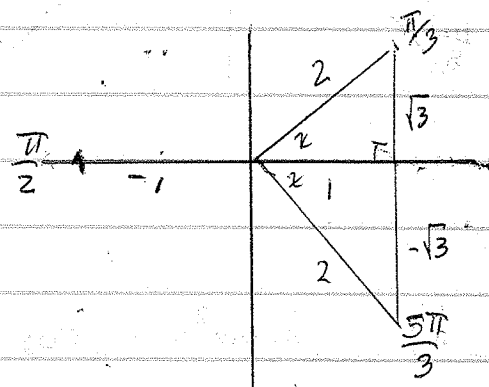
$$\boxed{x = \pi/3, 2\pi/3}$$

b) $2\cos^2 x + \cos x - 1 = 0$

$$(2\cos x - 1)(\cos x + 1) = 0$$

$$\cos x = \frac{1}{2}, \quad \cos x = -1$$

$$x = \frac{\pi}{3}, \frac{5\pi}{3}, \pi$$



c) $-\cot(2x) = 2 + \tan(2x)$

$$\frac{-1}{\tan(2x)} = 2 + \tan(2x)$$

multiply by $\tan(2x)$

$$-1 = 2\tan(2x) + \tan^2(2x)$$

$$0 = \tan^2(2x) + 2\tan(2x) + 1 \quad \leftarrow \begin{array}{l} \text{factors} \\ \text{as } (a+1)(a+1) \end{array}$$

$$0 = [\tan^{-1}(2x) + 1]^2$$

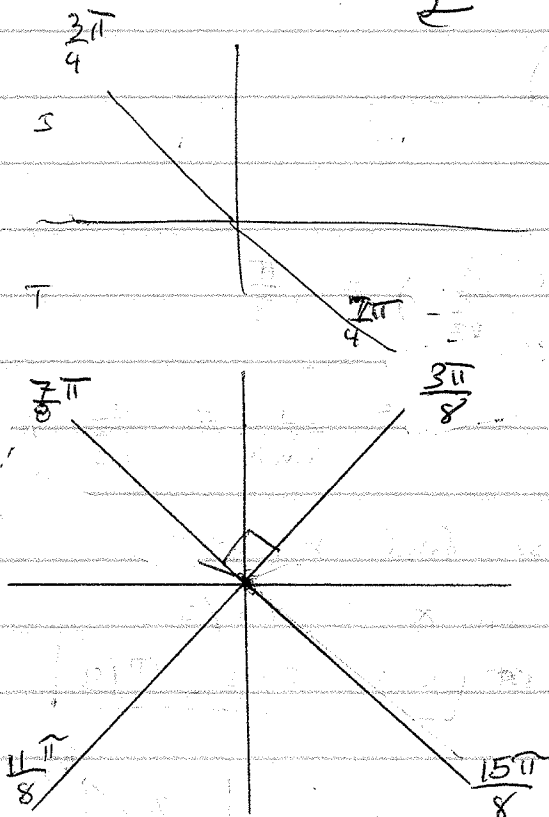
$$\tan^{-1}(2x) = -1$$

— Find what angle $2x$ gives $\tan 2x = -1$

$$\tan^{-1}(1) = 2x \quad \text{To isolate } x$$

$$\frac{\tan^{-1}(1)}{2} = x$$

Now find where $\tan^{-1}$



$$\tan(2x) = -1 \quad \text{at}$$

$$2x = \frac{3\pi}{4} + \frac{7\pi}{4}$$

$$\text{Hence } x = \frac{3\pi}{8}, \frac{7\pi}{8}$$

But we seek all solns.

on $[0, 2\pi)$, so we

have 2 more answers.

Since we cut angle in 2.

You could check to see that each of these gives

$$\tan(2x) = -1$$

$$\#1 \&) \quad 2\sin^2 x - \cos 2x = 0$$

Try double angle for $\cos 2x$

$$2\sin^2 x - (\cos^2 x - \sin^2 x) = 0$$

$$2\sin^2 x + \sin^2 x - \cos^2 x = 0$$

$$3\sin^2 x - \cos^2 x = 0$$

Now use $\cos^2 x = 1 - \sin^2 x$

$$3\sin^2 x - (1 - \sin^2 x) = 0$$

$$3\sin^2 x + \sin^2 x = 1$$

$$4\sin^2 x = 1$$

$$\sin^2 x = \frac{1}{4}$$

$$\sin x = \pm \frac{1}{2}$$

This soln lies in all quadrants.

$$x = \frac{\pi}{6}, \frac{5\pi}{6} \quad \text{for } \sin x = \frac{1}{2}$$

$$x = \frac{7\pi}{6}, \frac{11\pi}{6} \quad \text{for } \sin x = -\frac{1}{2}$$

#1e $\tan^2(2x) + 5 = 3 \sec^2(2x)$ Take advantage
 $\sec^2(2x) - 1 + 5 = 3 \sec^2(2x)$ of $\tan^2 \alpha + 1 = \sec^2 \alpha$

$$0 = 3 \sec^2(2x) - \sec^2(2x) - 4$$

$$0 = 2 \sec^2(2x) - 4$$

$$0 = \sec^2(2x) - 2$$

$$\sec^2(2x) = 2$$

Algebra

$$\sec(2x) = \pm \sqrt{2} \rightarrow \text{Hey! } \sec(2x) = \frac{1}{\cos(2x)}$$

and we know $\pm 1/\sqrt{2}$

is $\cos(\frac{\pi}{4}) + \cos(\frac{7\pi}{4})$

$$\sec(2x) = \pm \sqrt{2} \leftarrow \cos(\frac{3\pi}{4}) + \cos(\frac{5\pi}{4})$$

$$2x = \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}$$

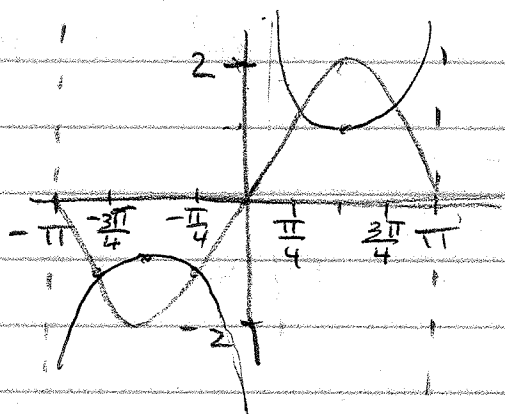
$$\text{and } \boxed{x = \frac{\pi}{8}, \frac{3\pi}{8}, \frac{5\pi}{8}, \frac{7\pi}{8}}$$

We still have another " π to go" in $[0, 2\pi)$

Generate the rest by adding $\frac{2\pi}{8}$ successively.

$$\boxed{x = \frac{9\pi}{8}, \frac{11\pi}{8}, \frac{13\pi}{8}, \frac{15\pi}{8}}$$

#2) a) $2 \sin x = \csc x$



Before we try to do this "algebraically", consider the graphs of $\csc x + 2 \sin x$. Where do they intersect.

4 pts of intersection; Do algebra

Given: $2 \sin x = \csc x$

Set = zero: $2 \sin x - \csc x = 0$

Use def of $\csc x$: $2 \sin x - \frac{1}{\sin x} = 0$

LCD: $2 \sin^2 x - 1 = 0$

Solve: $2 \sin^2 x = 1 \rightarrow \sin x = \pm \sqrt{2}/2$

$x = \pm \pi/4, \pm 3\pi/4$

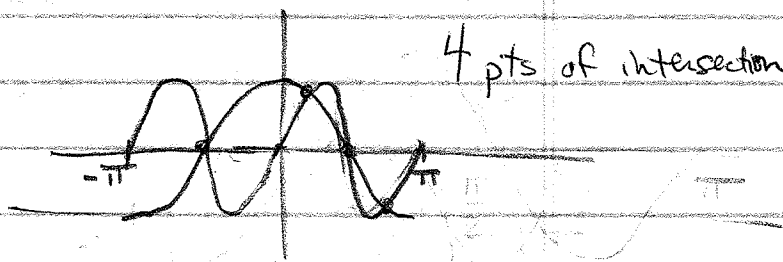
c) $\sin x \cos x = \frac{1}{2} \cos x$

$2 \sin x \cos x = \cos x$

$\sin 2x = \cos x$

Good for analyzing by graph

but to solve, leave as $\sin x \cos x = \frac{1}{2} \cos x$.



4 pts of intersection

$2 \sin x \cos x = \cos x$

$2 \sin x \cos x - \cos x = 0$

$\cos x (2 \sin x - 1) = 0$

$\cos x = 0, 2 \sin x - 1 = 0$

$\Rightarrow x = \frac{\pi}{2}, -\frac{\pi}{2}, \frac{\pi}{6}, \frac{5\pi}{6}$
on $[-\pi, \pi]$

