Sec 8.8

#1) Requires sum-to-product formula, so you may skip.

#2) Show $\sin(\alpha + 2\pi) = \sin\alpha$ using sum formula:

$$\sin(\alpha + 2\pi) = \sin\alpha \cos(2\pi) + \sin(2\pi)\cos\alpha$$
$$= \sin\alpha \cdot 1 + 0 \cdot \cos\alpha$$
$$= \sin\alpha \quad ✓$$

#3) Verify $(\sin x + \cos x)^2 = 1 + \sin(2x)$

On left: $\sin^2 x + 2\sin x\cos x + \cos^2 x$
$$= 1 + 2\sin x \cos x$$
$$= 1 + \underbrace{\sin 2x}$$
(Double angle formula)

#4) Evaluate using sum + diff formulas:

a) $\cos^2\left(\frac{\pi}{8}\right) - \sin^2\left(\frac{\pi}{8}\right) = \cos\left(2 \cdot \frac{\pi}{8}\right) = \cos\left(\frac{\pi}{4}\right)$
from double angle formula

$$\boxed{\cos\left(\frac{\pi}{4}\right) = \sqrt{2}/2} \quad //$$

b) $\sin\left(\frac{\pi}{5}\right)\cos\left(\frac{3\pi}{10}\right) + \cos\left(\frac{\pi}{5}\right)\sin\left(\frac{3\pi}{10}\right)$

from sum formula $= \sin\left(\frac{\pi}{5} + \frac{3\pi}{10}\right)$
$$= \sin\left(\frac{5}{10}\pi\right)$$
$$= \sin(\pi/2) = \boxed{1} \quad //$$

c) $[\tan(x+\pi)][\tan(x-\pi)] + 1$

Use Sum + Diff formula for tan:

$$[\tan(\alpha+\beta)][\tan(\alpha-\beta)]$$

$$= \frac{\tan\alpha + \tan\beta}{1-\tan\alpha\tan\beta} \cdot \frac{\tan\alpha - \tan\beta}{1+\tan\alpha\tan\beta}$$

Sub x + π for α + β:

$$\frac{\tan x + \tan\pi}{1-\tan x\tan\pi} \cdot \frac{\tan x - \tan\pi}{1+\tan x\tan\pi}$$

$$= \frac{\tan^2 x - \tan^2\pi}{1-\tan^2 x\tan^2\pi}$$

$$= \frac{\tan^2 x - 0}{1-(\tan^2 x)0} = \frac{\tan^2 x}{1} = \tan^2 x$$

Given that $\underline{1}$ is added to this:

$$\tan^2 x + 1 = \boxed{\sec^2 x} \text{ from identity } //$$

#4d) $\dfrac{\tan\left(\frac{\pi}{5}\right) - \tan\left(\frac{\pi}{30}\right)}{1 + \tan\left(\frac{\pi}{5}\right)\tan\left(\frac{\pi}{30}\right)} = \tan\left(\frac{\pi}{5} - \frac{\pi}{30}\right)$

$\uparrow$ from diff formula for tangent

$$\tan\left(\frac{\pi}{5} - \frac{\pi}{30}\right) = \tan\left(\frac{6\pi - \pi}{30}\right)$$

$$= \tan\left(\frac{5\pi}{30}\right) = \tan\left(\frac{\pi}{6}\right) = \boxed{\frac{1}{\sqrt{3}}} \quad //$$

#6 $\sin(3x) \overset{?}{=} 3\sin x - 4\sin^3 x$

Note: $\sin(3x) = \sin[(2x) + (x)]$

Let's try using 3 ~~two~~ formulas:

(1) $\sin(2x) = 2\sin x\cos x$, (3) $\sin(x+y) =$
(2) $\cos(2x) = \cos^2 x - \sin^2 x$, $\sin x\cos y + \sin y\cos x$

Then:

$$\sin(3x) = \sin[(2x) + x]$$

$$= \sin(2x)\cos(x) + \sin(x)\cos(2x)$$
$$= 2\sin x\cos x\cos x + (\sin x)(\cos^2 x - \sin^2 x)$$
$$= 2\sin x\cos^2 x + \sin x\cos^2 x - \sin^3 x$$
$$= 2\sin x(1-\sin^2 x) + \sin x(1-\sin^2 x) - \sin^3 x$$
$$= 2\sin x - 2\sin^3 x + \sin x - \sin^3 x - \sin^3 x$$
$$= 3\sin x - 4\sin^3 x \quad //$$

Wow! I did it! And so can you ☺

#8) a) $\cos(105^\circ) = \cos(60^\circ + 45^\circ)$

$= \cos 60^\circ \cos 45^\circ - \sin 60^\circ \sin 45^\circ$

$= \frac{1}{2} \cdot \frac{\sqrt{2}}{2} - \frac{\sqrt{3}}{2} \cdot \frac{\sqrt{2}}{2}$

$= \frac{\sqrt{2} - \sqrt{6}}{4}$ //

b) $\sin\left(\frac{11\pi}{12}\right) = \sin\left(\frac{3\pi}{12} + \frac{8\pi}{12}\right)$

$= \sin\left(\frac{\pi}{4} + \frac{2\pi}{3}\right)$

$= \sin\left(\frac{\pi}{4}\right)\cos\left(\frac{2\pi}{3}\right) + \sin\left(\frac{2\pi}{3}\right)\cos\left(\frac{\pi}{4}\right)$

$= \left(\frac{\sqrt{2}}{2}\right)\left(-\frac{1}{2}\right) + \left(\frac{\sqrt{3}}{2}\right)\left(\frac{\sqrt{2}}{2}\right)$

$= \frac{-\sqrt{2}}{4} + \frac{\sqrt{6}}{4} = \frac{\sqrt{6} - \sqrt{2}}{4}$ //

c) $\sin(195^\circ) = \sin(150^\circ + 45^\circ)$

$= \sin 150^\circ \cos 45^\circ + \sin 45 \cos 150$

$= \sin 30^\circ \cos 45 + (\sin 45)(-\cos 30)$

$= \frac{1}{2} \cdot \frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2} \cdot \left(-\frac{\sqrt{3}}{2}\right)$

$= \frac{\sqrt{2}}{4} - \frac{\sqrt{6}}{4} = \frac{\sqrt{2} - \sqrt{6}}{4}$