Sec 9.2

#3 A) Ellipse $16x^2+64x+9y^2-54y+1=0$

$$\to 16(x^2+4x+4)+9(y^2-6y+9)=-1+64+81$$

$$16(x+2)^2+9(y-3)^2=144 \leftarrow$$

$$\frac{16}{144}(x+2)^2+\frac{9}{144}(y-3)^2=1 \leftarrow \text{want } 1$$

$$\frac{(x+2)^2}{9}+\frac{(y-3)^2}{16}=1$$

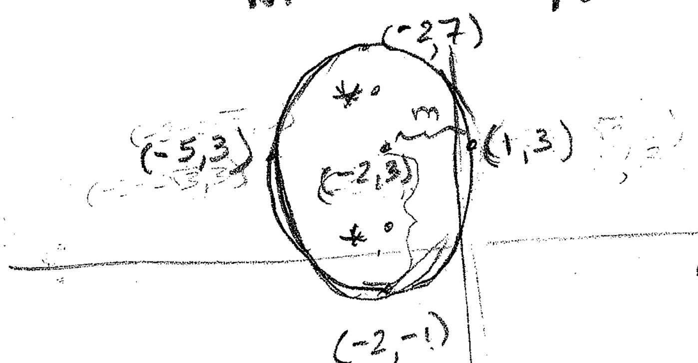
$$\frac{(x-h)^2}{m^2}+\frac{(y-k)^2}{M^2}=1$$

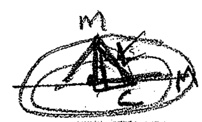
$m^2=9$, $m=3$

$M^2=16$, $M=4$

$2M=D=8$

center $(-2,3)$





$m^2+c^2=\left(\frac{D}{2}\right)^2$

$\boxed{m^2+c^2=M^2}$

$c=\sqrt{M^2-m^2}$

$=\sqrt{16-9}$

$=\sqrt{7}$

* foci: $(-2, 3+\sqrt{7})$ and $(-2, 3-\sqrt{7})$

Sec 9.3

#1 d) $6x^2-4y^2=12$

2 squared terms, subtraction $\Rightarrow$ hyperbola

$$\frac{6x^2}{12}-\frac{4y^2}{12}=1$$

$$\frac{x^2}{2}-\frac{y^2}{3}=1$$

$a^2=2$ $b^2=3$

$a=\sqrt{2}$, $b=\sqrt{3}$

$a^2+b^2=c^2$

$2+3=5$

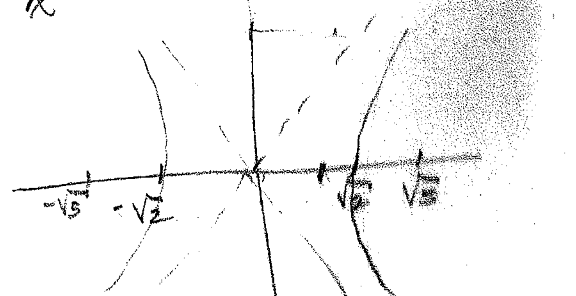
$c=\sqrt{5}$

$D=2\sqrt{2}$

The vertices + foci are on x-axis since the positive term is x^2

$$y=\pm\frac{\sqrt{3}}{\sqrt{2}}x$$

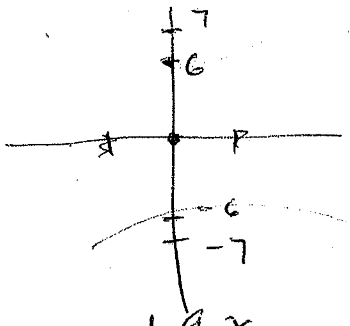
$$=\pm\sqrt{3/2}\,x$$



$\rightarrow$ F $(0,7), (0,-7)$ "diff dist" $D = 12 \leftarrow$

#2 b) $c = 7$ since foci are $(0,7), (0,-7)$

Orientation is vertical since foci are on y-axis center $(0,0)$



hyperbola (recall $D = 2M$ for ellipse)

$D = 12 = \boxed{2a}$, so $a = 6$

$$\left.\begin{aligned} a^2 + b^2 &= c^2 = 7^2 \\ 36 + b^2 &= 49 \\ b &= \sqrt{13} \end{aligned}\right\} \quad \frac{y^2}{36} - \frac{x^2}{13} = 1$$

$y = \pm \frac{a}{b} x$
$= \pm \frac{6}{\sqrt{13}} x$

note: x & y change places but a & b remain

#2c)

Given
$y = \pm \frac{2}{1} x$
F $(10,0)$

$y = 2x = \frac{b}{a}x$ or $\frac{a}{b}x$

But focus is $(10,0)$, hence the orientation is horizontal, so vertices are on x-axis

Thus, the slope of the asymptotes is $\pm \frac{b}{a} x \leftarrow$

Since $\frac{b}{a} = \frac{2}{1}$ then $b = 2a$

$\rightarrow c^2 = a^2 + b^2 = a^2 + (2a)^2 = 5a^2$

But $c = 10$ so $c^2 = 100 = 5a^2$

$\Rightarrow 20 = a^2$

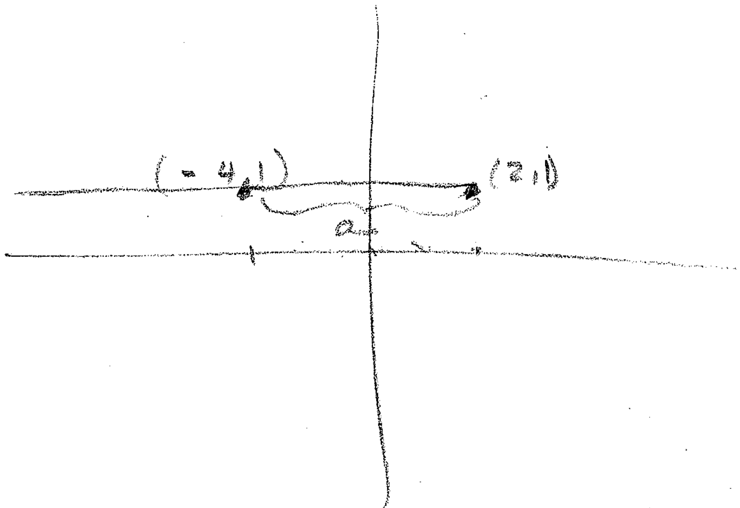
$80 = b^2$

$$\boxed{\frac{x^2}{20} - \frac{y^2}{80} = 1}$$

Notice that the relative magnitudes of a & b do not affect orientation

Finding equations from information

2) Center $(-4,1)$ Vertex $(2,1)$ Asymptote slope: $\frac{1}{3}$



Since the vertex + center have same y-coordinate, this is a horizontally oriented hyperbola.

$$\frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{b^2} = 1$$

$$\frac{(x-(-4))^2}{a^2} - \frac{(y-1)^2}{b^2} = 1$$

Since $y = \pm \frac{b}{a}x = \pm \frac{1}{3}x$, the ratio of $\frac{b}{a}$ is $\frac{1}{3}$.

We know $a = 2-(-4)$ from the vertex - center x-coord., that is, $\boxed{a = 6}$ so

$$\frac{b}{a} = \frac{b}{6} = \frac{1}{3} \text{ (given)}$$

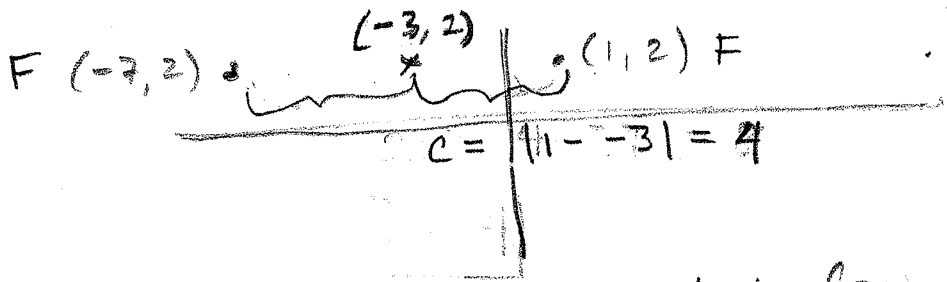
Hence, $b = 2$. The eqn. is $\boxed{\frac{(x+4)^2}{36} - \frac{(y-1)^2}{4} = 1}$

#15b) Foci are $(1,2)$, $(-7,2)$

$D = 2a = 6$ (Don't forget the relationship btwn difference dist and length a)

$$\boxed{a = 3}$$

Since we know the length from F_1 to F_2, we know the center, halfway between.



$$\boxed{(h,k) = (-3,2)}$$

The value c is dist from center to F_1 as shown: $c = |1 - -3| = 4$ $\boxed{c = 4}$

So far: $$\frac{(x+3)^2}{3^2} - \frac{(y-2)^2}{b^2} = 1$$

We still need b:

$$a^2 + b^2 = c^2$$

$$b = \sqrt{c^2 - a^2} = \sqrt{16 - 9}$$

$$= \sqrt{7}$$

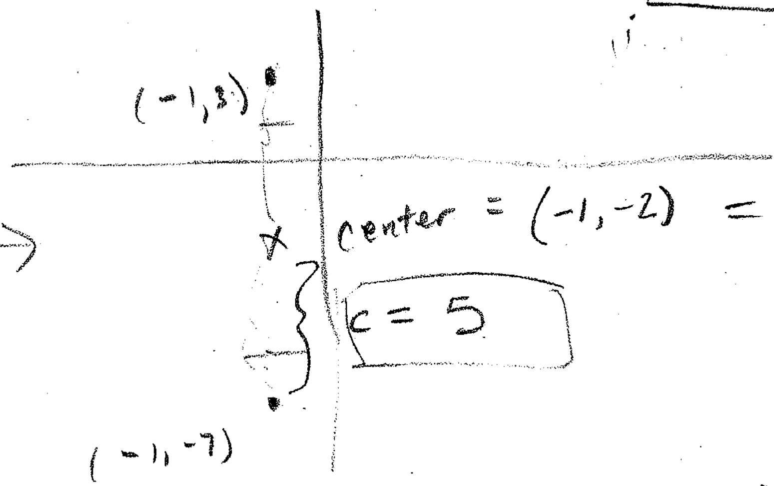
<u>Eqn</u> $$\frac{(x+3)^2}{9} - \frac{(y-2)^2}{7} = 1$$

#5 e) Focii are $(-1,3)$ $(-1,-7)$

Diff dist $D = 2\sqrt{11}$

$D = 2a = 2\sqrt{11} \Rightarrow \boxed{a = \sqrt{11} \text{ - dist from center to vertex}}$

Vert. oriented $\longrightarrow$



center $= (-1,-2) = (h,k)$

$\boxed{c = 5}$

So ~~far~~ $\dfrac{(y - {}^-2)^2}{(\sqrt{11})^2} - \dfrac{(x - {}^-1)^2}{b^2} = 1$

$b^2 = c^2 - a^2 = 25 - 11 = 14$

$b = \sqrt{14}$

$\therefore \dfrac{(y+2)^2}{11} - \dfrac{(x+1)^2}{14} = 1$