Lecture notes 10/28

Though Ch. 10 begins with exponential, we looked at the log fcn. first to define

$$\log_a x = y \iff x = a^y$$
$$a^y = x$$

base logarithm antilog

"The logarithm of x in base a is y."

So, the log is y. The antilog is x.

Def A logarithm is a power that a given base is raised to in order to obtain a given antilog.

Ex $\log_2 4 = y \to 4 = 2^y$
$\to y = 2$

$\log_{10} \frac{1}{100} = y \to \frac{1}{100} = 10^y$

$\to y = -2$

$\log_2 1 = y \to 1 = 2^y$
$\to y = 0$

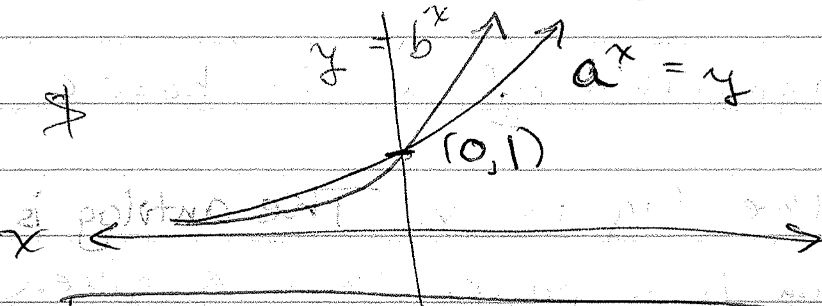
In fact, for any base a, $\log 1 = 0$.

Later we'll see... (**Q** $\log_7 3 = y \to 3 = 7^y$

Rule $\log_7 3 = \dfrac{\log_{10} 3}{\log_{10} 7}$ ← Table for base 10)

$\underline{\text{Fcn}}$: $f(x) = \log_a x = y \xrightarrow{\text{means}} a^y = x$

$\underline{\text{Inverse}}$: $a^x = y = f^{-1}(x)$ $\underline{\text{Exponential fcn.}}$



$a^x = y$ $\quad a < b$

$a^0 = 1$

y - intercept

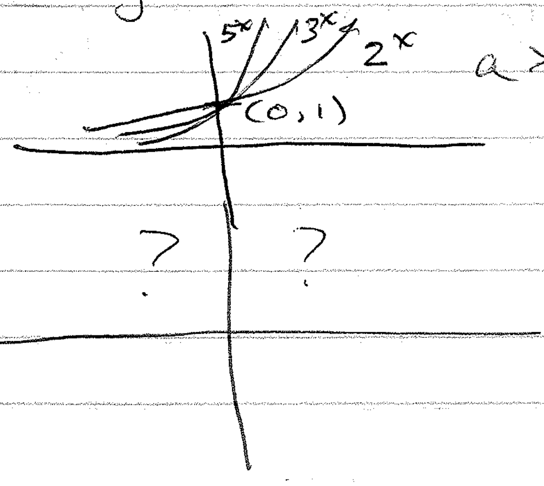
$\boxed{\text{A word about base } a\text{: Consider } \boxed{0 < a < 1} \text{ or } \boxed{a > 1}}$

- because 0 as a base gives 0^0 (undefined)
  and $0^x = 0 \cdot 0 \cdot \cdots 0 = 0$
  uninteresting
  useless
- because 1
  as a base gives $1^x = 1 \cdot 1 \cdots 1 = 1$
  which ~~the~~ is the constant fcn. $y = 1$ –
  also useless + uninteresting!

So far, $a \in \mathbb{Z}^+$, $a \in \mathbb{Q}^+$ rational

But we'll also look at $a \in \mathbb{Q}'$ irrational

- In fact a is any positive $\neq 1$.
- $a > 1$ gives the obvious graphs



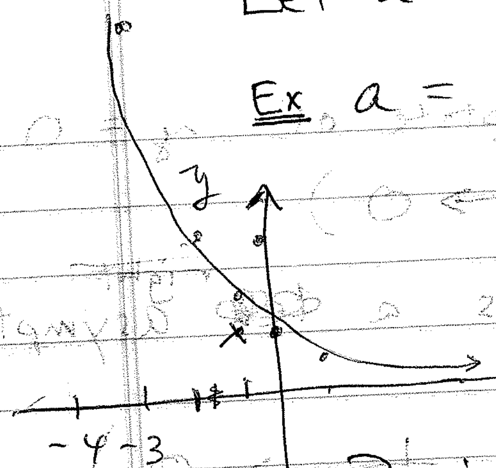
$a > 1$

- $0 < a < 1$
  looks like
  What?

$0 < a < 1$

Let a be < 1:

<u>Ex</u> $a = \frac{1}{2}$



x	$y = a^x = \left(\frac{1}{2}\right)^x$
-4	$\left(\frac{1}{2}\right)^{-4} = 2^4 = 16$
-3	$\left(\frac{1}{2}\right)^{-3} = 2^3 = 8$
-2	4
-1	2
0	1
1	$1/2$
2	$(1/2)^2 = 1/4$
3	$\left(\frac{1}{2}\right)^3 = 1/8$

domain

Reminds us of a^x where $a = 2$.

~~It's the reflection~~ $y = a^x$.

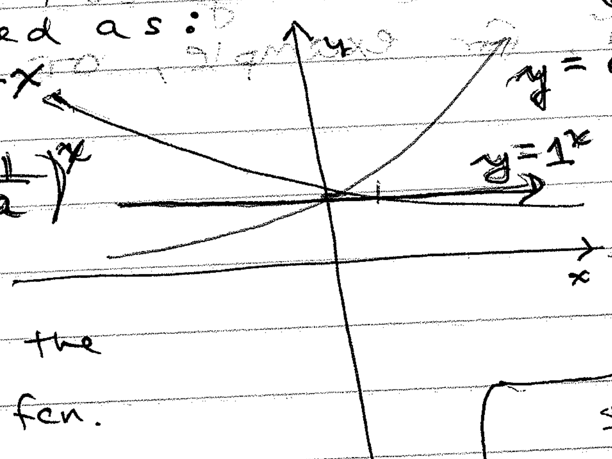
- Domain of $f(x) = a^x$ is $\mathbb{R}$
- Range of $f(x) = a^x$ is $y > 0$

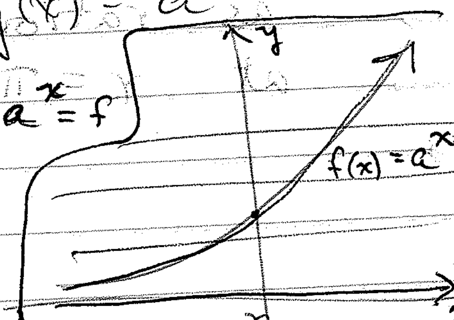
Apparently, when $0 < a < 1$, the graph flips on the y-axis and you get

(for $a > 1$) $\boxed{y = \left(\frac{1}{a}\right)^x = a^{-x}}$

- So, $f(x) = a^x$ and $g(x) = a^{-x}$ are graphed as:

$g = y = a^{-x} = \left(\frac{1}{a}\right)^x$





$y = 1^x$ is the constant fcn. $y = 1$.

$y = a^x$ are 1-1 so they make easy transformation

<u>Final note today:</u>

$f(x) = a^x$ is one-one

(passes horizontal line test)

- It has a left asymptote of $y = 0$:
  (as $x \to -\infty$, $a^x \to 0$)
- Also, $g(x) = a^{-x}$ has a ~~left~~ right asymptote of $y = 0$:
  (as $x \to \infty$, $a^{-x} = \frac{1}{a^x} \to 0$)

- All fcns $f(x) = a^x$ have a y-int of $(0,1)$ and no x-int.

- Tuesday: We'll discuss why $y = a^x$ is graphed as we've done for all real x.
  That is x can be irrational as well as rational (so $y = 2^x$ has a value at $x = \pi$, for example, or $x = \sqrt{5}$)