

① Mental math: You should be able to answer w/o calculator problems like:

$$\bullet (-15)(2) + 6$$

$$\bullet \frac{1}{5} + \frac{8}{3}$$

$$\bullet 1428 \div 3$$

$$\bullet \frac{(-2)(-2)(-2)}{-1}$$

$$\bullet 20 - 14.60$$

(Throwing in a decimal.)

② For any real numbers a, b, c , the following axioms hold:

$$a = a \quad (\text{reflexive property})$$

$$a = b \rightarrow b = a \quad (\text{symmetric property})$$

$$a = b, b = c \rightarrow a = c \quad (\text{transitive property})$$

• Addition + subtraction are inverse ^{operations} ~~properties~~
 Division + multiplication are inverse ops.

$$\bullet a \cdot 0 = 0 \cdot a = 0 \quad \text{Zero property of } \otimes$$

$$\frac{a}{0} \text{ is undefined,} \quad \frac{0}{a} = 0 \text{ for } a \neq 0$$

$$\bullet a \cdot 1 = 1 \cdot a = a$$

$$a + 0 = 0 + a = a$$

multiplicative identity
 additive identity

- ③
- Horizontal lines have a slope of zero.
 - Vertical lines have an undefined slope.
 - The equation of a horizontal line through a pt. (a, b) is $y = b$.
 - The equation of a vertical line through a pt. (a, b) is $x = a$.
 - The equation of the x -axis is $y = 0$.
 - The equation of the y -axis is $x = 0$.

④ The equation of a line through the pt $(-2, 6)$ with a slope of $-1/3$ is found in 3 steps:

(i) $6 = -\frac{1}{3} \cdot -2 + b$
 $y = m x + b$

from $y =$
 (ii) solve for b : $6 = \frac{2}{3} + b \rightarrow b = \frac{16}{3}$ (why?)

(iii) Substitute $b + m$ into $y = mx + b$

$$\boxed{y = -\frac{1}{3}x + \frac{16}{3}}$$

To continue to the standard form of line, multiply by the LCD:

(iv) $3y = -x + 16 \rightarrow \boxed{x + 3y = 16}$

⑤ Graph a line from an equation in the standard form. $Ax + By = C$.

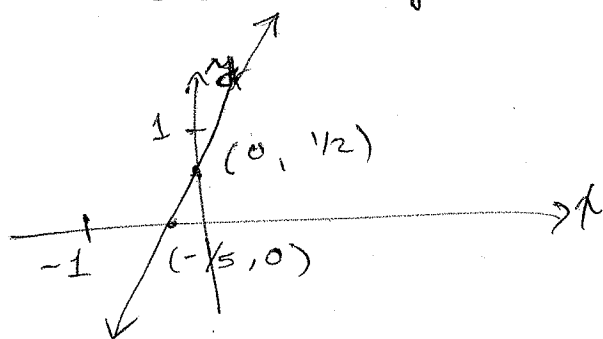
The best way to do this is to find 2 pts, the x & y intercepts. That is,
let $y=0$ + solve for x -intercept;
let $x=0$ + solve for y -intercept.

Ex Graph $5x - 2y = -1$

The two pts we seek are the x - & y -intercepts

$$5x - 2(0) = -1 \rightarrow x = -1/5, (-1/5, 0)$$

$$5(0) - 2y = -1 \rightarrow y = 1/2, (0, 1/2)$$



$$m = \frac{1/2 - 0}{-1/5 - 0} = \frac{1/2}{-1/5} = -5/2$$

Make sure you can find in reduced form like this a slope.

⑥ Where do the lines $3x - y = 7$ and $x + y = 0$ intersect?

Substitution

From $x + y = 0$, $y = -x$, $3x - (-x) = 7$, $4x = 7$,
 $x = 7/4$

Answer $\rightarrow (7/4, -7/4)$ $y = -7/4$

⑥ Where do the lines $5x - 2y = 8x - 1$ and $6x + y = -3y + 2$ intersect?

By elimination (first bring variable terms to one side of equation):

$$\begin{array}{rcl} 3x + 2y & = & 1 \\ 6x + 4y & = & 2 \end{array} \quad \xrightarrow{\times(-2)} \quad \begin{array}{rcl} -6x - 4y & = & -2 \\ 6x + 4y & = & 2 \\ \hline 0 & = & 0 \end{array}$$

These intersect everywhere. One line lies on the other.

⑦ Solve by Cramer's Rule the system of planes

$$\begin{cases} 2x + z = 1 \\ -z = 2 \\ -x + y = -2 \end{cases}$$

Set up the matrices A, A_x, A_y, A_z

$$A = \begin{pmatrix} 2 & 0 & 1 \\ 0 & 0 & -1 \\ -1 & 1 & 0 \end{pmatrix}, \quad A_x = \begin{pmatrix} 1 & 0 & 1 \\ 2 & 0 & -1 \\ -2 & 1 & 0 \end{pmatrix}$$

$$A_y = \begin{pmatrix} 2 & 1 & 1 \\ 0 & 2 & -1 \\ -1 & -2 & 0 \end{pmatrix}, \quad A_z = \begin{pmatrix} 2 & 0 & 1 \\ 0 & 0 & 2 \\ -1 & 1 & -2 \end{pmatrix}$$

Hence

$$D = 2(0 - (-1)) + 0 + 1(0 - 0) = 2$$

$$D_x = 1(0 - (-1)) + 0 + 1(2 - 0) = 3$$

$$D_y = 2(0 - 2) + 1(0 - 1) + 1(0 - 2) = -3$$

$$D_z = 2(0 - 2) + 0 + 1(0 - 0) = -4$$

$$x = \frac{D_x}{D} = \frac{3}{2}, \quad y = \frac{D_y}{D} = -\frac{3}{2}, \quad z = \frac{D_z}{D} = -2$$

That is, the planes intersect at a unique pt (like the corner of a room)

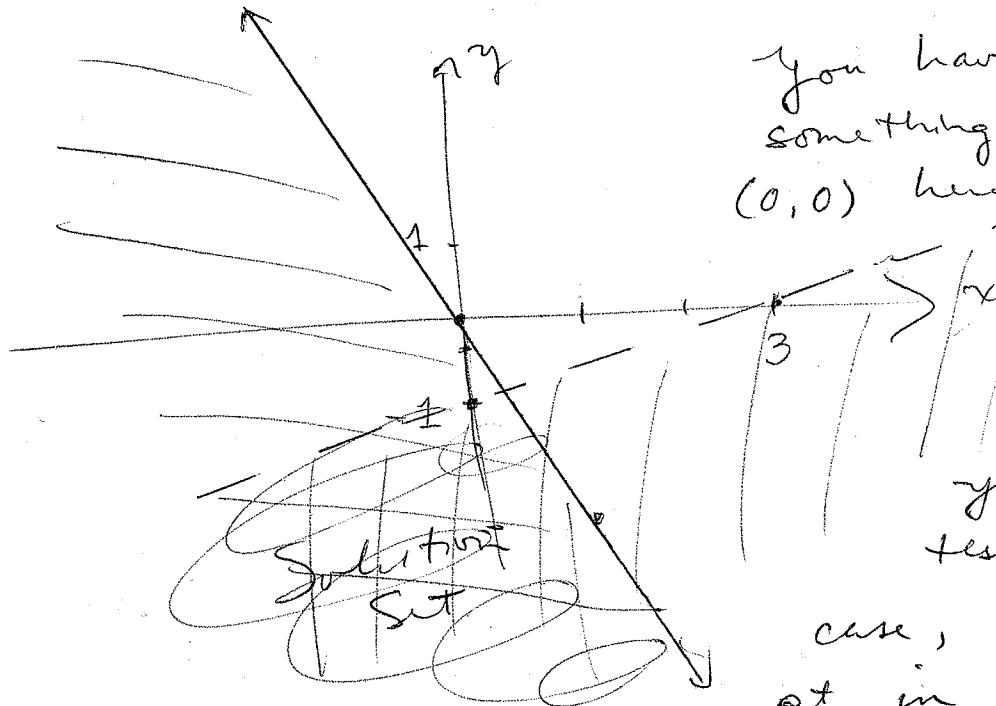
$$\left(\frac{3}{2}, -\frac{3}{2}, -2\right)$$

Check! I got the first 2 equations checking, but the third does not!

On the test we'll use similar (small) coefficients so we arrive fast at solution.

8. Graph the system of inequalities

$$\begin{aligned} x - 3y &> 3 & \longrightarrow & y < \frac{1}{3}x - 1 \\ 2x + y &\leq 0 & \longrightarrow & y \leq -2x \end{aligned}$$



You have to check something other than (0,0) here because there are other regions you need to test. In this case, find a pt in the soln set to test

It's good to choose a pt on the y -axis
below $y = -1$. Say $y = -2$, $x = 0$

Does $(0, -2)$ satisfy the two
inequalities?

$$x - 3y > 3 \longrightarrow 0 - 3(-2) = 6 > 3$$

$$2x + y \leq 0 \longrightarrow 0 - 2 = -2 \leq 0$$

checks!

⑨ #20 p. 266 Money

- 20. Investments** A trust administrator divides \$20,000 between two accounts. One account earns an annual simple interest rate of 3%, and the second account earns an annual simple interest rate of 7%. The total annual income from the two accounts is \$1200. How much is invested in each account?
\$5000 at 3%; \$15,000 at 7% [4.1C]

Unless otherwise noted, all

The money invested at 3% is x .

The money invested at 7% is y .

Since \$20,000 was invested

$$(\text{that is, } x + y = 20,000)$$

to earn \$1200

$$(\text{that is, } 0.03x + 0.07y = 1200)$$

The system of eqns. is

$$\left. \begin{array}{l} \textcircled{1} \quad x + y = 20,000 \\ \textcircled{2} \quad 3x + 7y = 120,000 \end{array} \right\}$$

Solve this
system
by substitution.