Math 108 Ch 4
Sec 4.1 HW

#1. One-to-oneness is defined ~~by~~ as a function's having a unique y for every x in its domain and a unique x for every y in its range.

But for a given fcn, we need the symbolic definition, since we can't check every $x \in \text{Dom } f$.

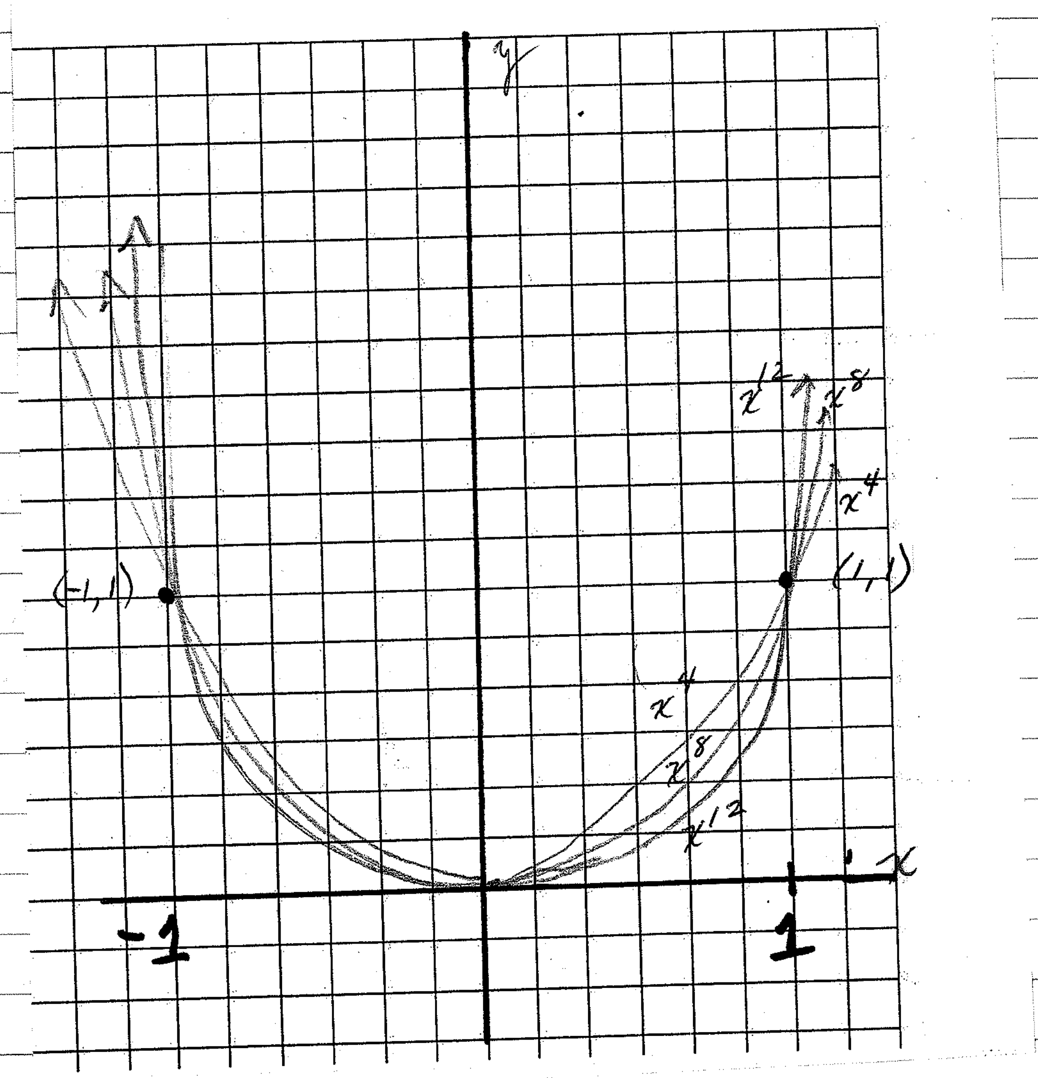
Thus, $f(x) = x$ is one-one because whenever $f(a) = f(b)$ for any $a, b \in \text{Dom } f$, it is true that $a = b$. (It's so easy it's hard):

$$f(a) = f(b)$$
$$a = b \checkmark$$

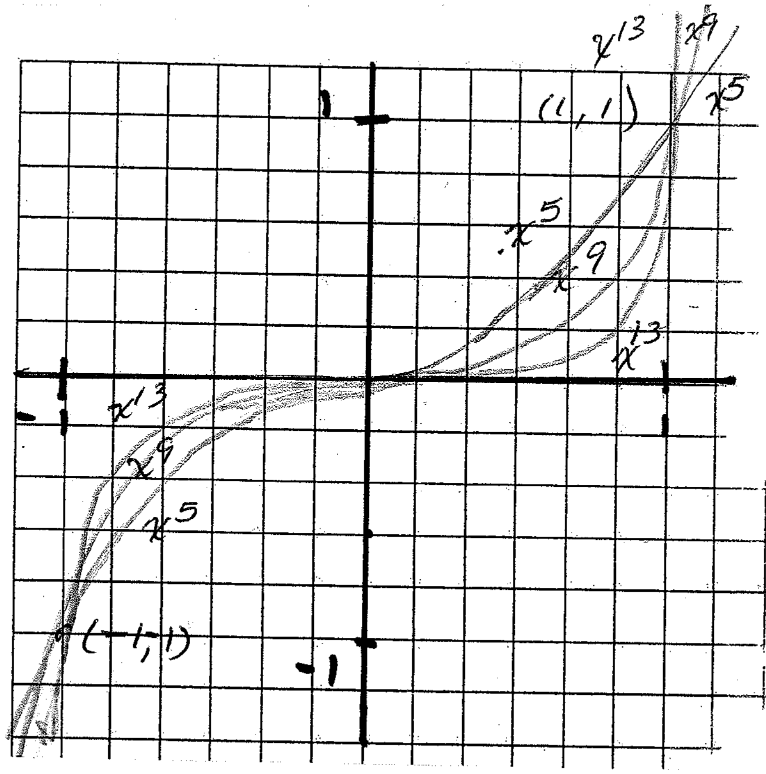
You could also say that when $f(a) \neq f(b)$, $a \neq b$. //

#(1) $f^{-1}(x) = x$ itself since
$y = x \longrightarrow x = y$ done!

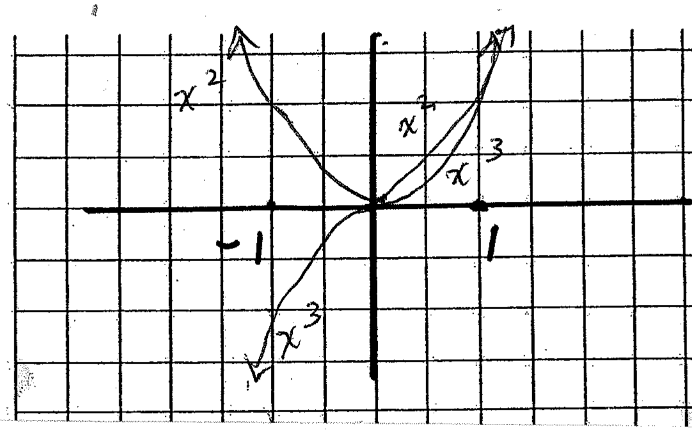
#2 A close up of $f = x^4$, $g = x^8$
& $h = x^{12}$



#3. A close up of $f = x^5$, $g = x^9$, $h = x^{13}$

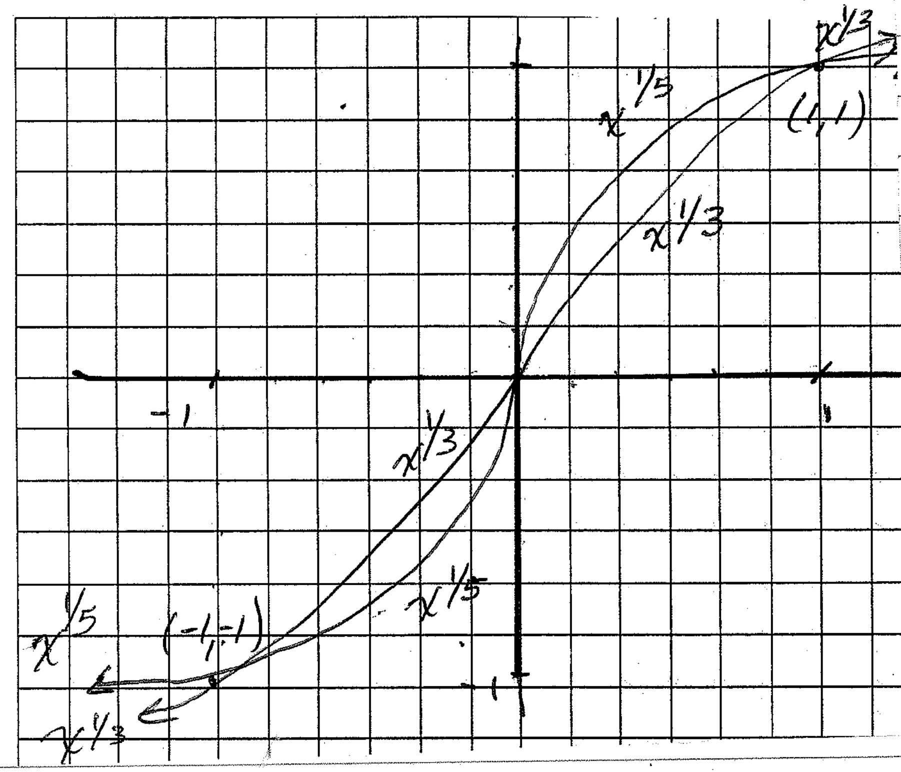


#4



#5 a) Just as $x^5 > x^3$ on $(0,1)$
$x^{1/5} < x^{1/3}$ on $(0,1)$

because we've inverted the graphs, so x + y trade places.

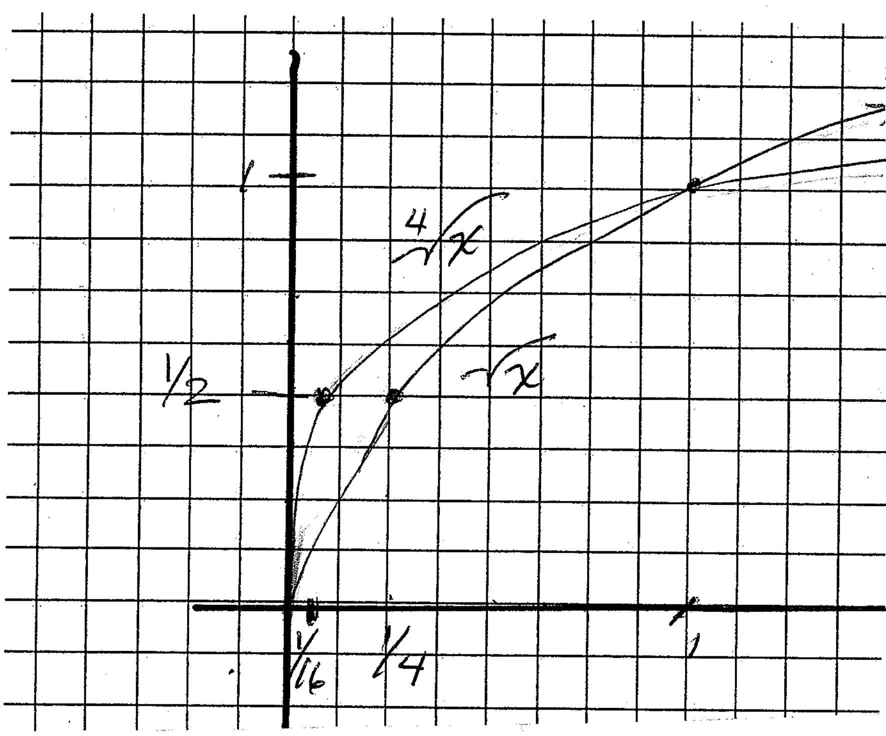


#5b) Dom $f + g$: $x \geq 0$

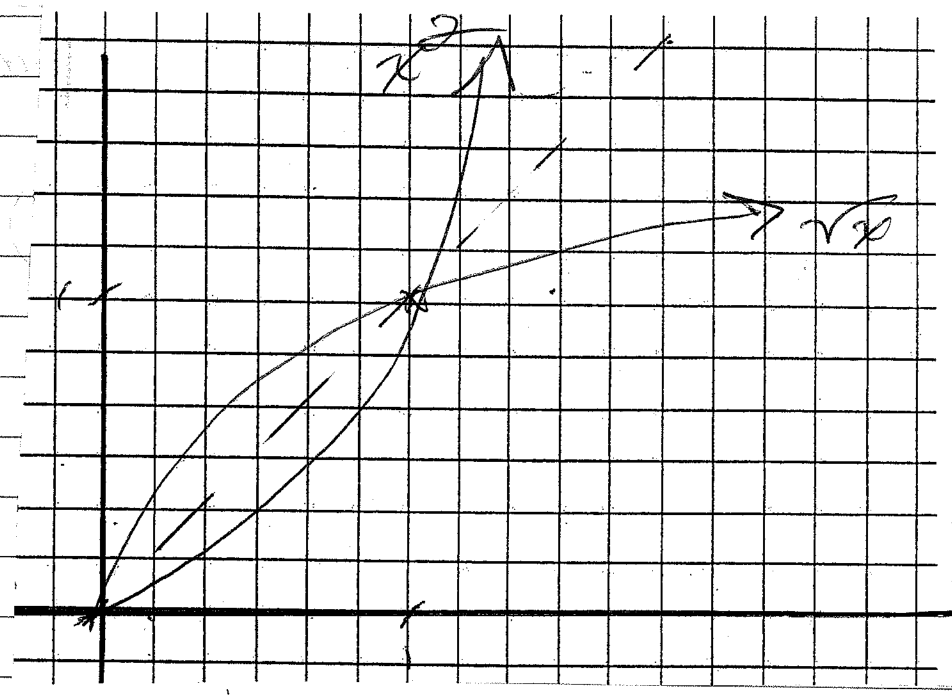
Consider that $\sqrt{1/4} = \sqrt[4]{1/16}$

that is, $1/2 = 1/2$

I've shown these pts on the graph to help position $x^{1/4} > x^{1/2}$ on $(0,1)$.



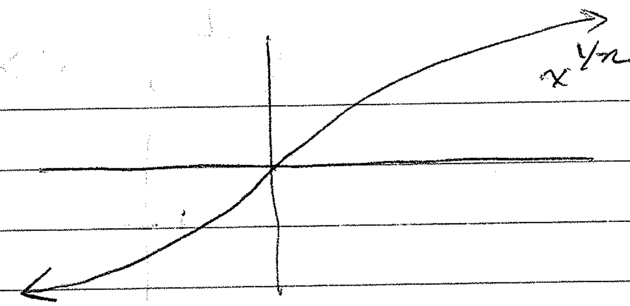
#5c) These should reflect across $y = x$ since they are inverses.



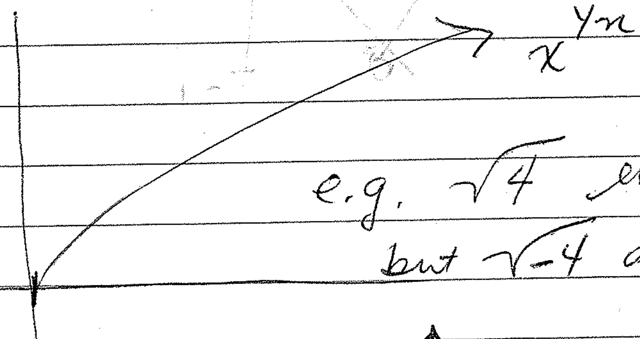
#6 For n odd, $f(x) = \sqrt[n]{x}$ is odd since $f(-x) = -f(x)$ (the odd root of a negative is negative, opposite in sign to the odd root of its positive absolute value.

e.g. $\sqrt[3]{8} = -\sqrt[3]{-8}$

Graphically:

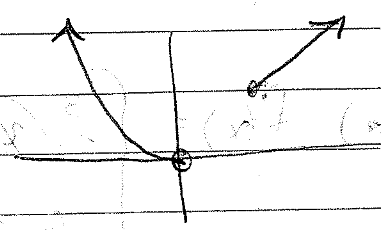


#7) For n even, $f(x) = \sqrt[n]{x}$ is neither odd nor even, since there is no $x < 0$ to consider.

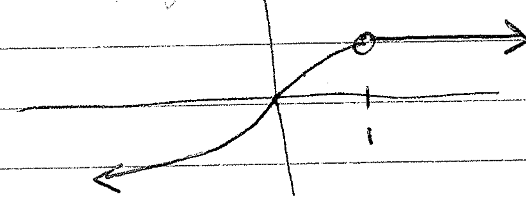


e.g. $\sqrt{4}$ exists but $\sqrt{-4}$ does not.

#8) (a) $f(x) = \begin{cases} x^2, & x < 0 \\ x, & x \geq 1 \end{cases}$

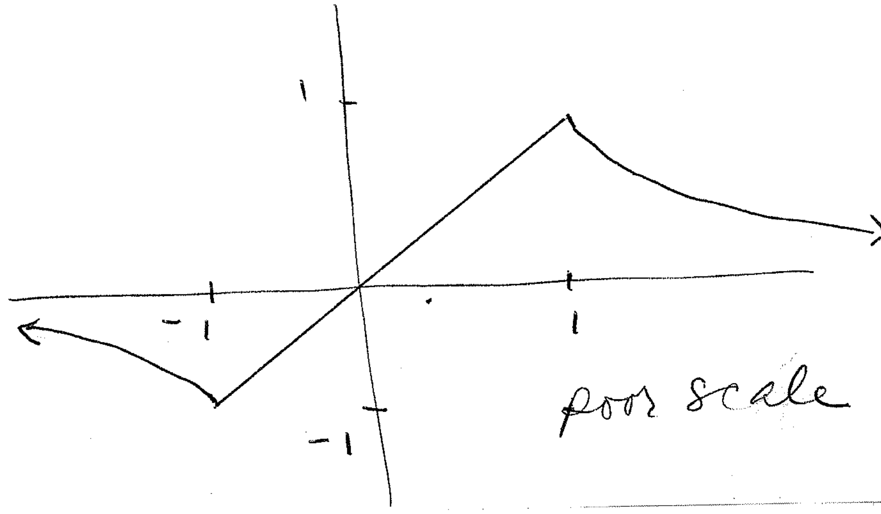


(b) $f(x) = \begin{cases} x^3, & x < 1 \\ 1, & x > 1 \end{cases}$



(c)

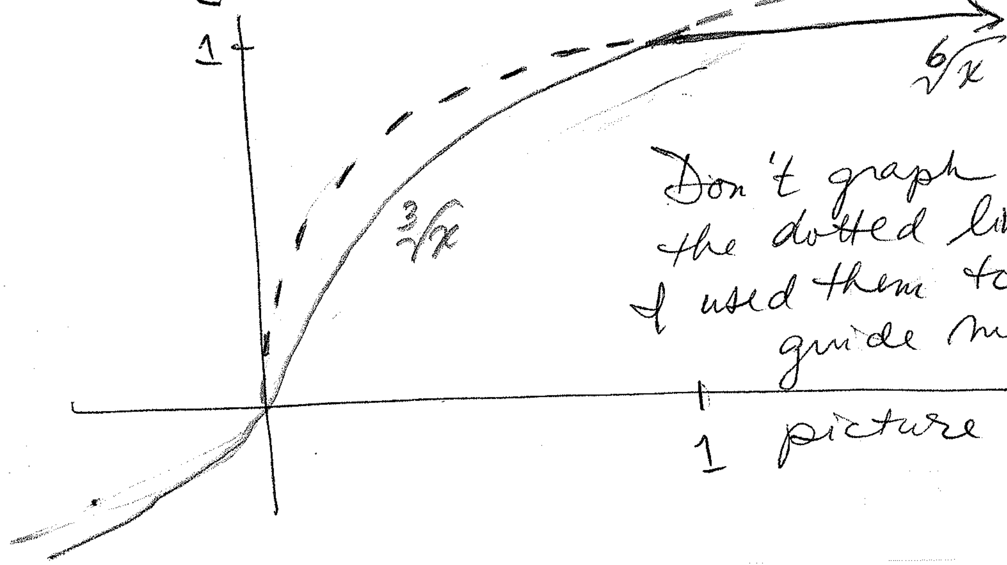
$$f(x) = \begin{cases} 1/x, & x < 1 \\ x, & -1 \le x \le 1 \\ 1/x, & x > 1 \end{cases}$$



poor scale

(d)

$$f(x) = \begin{cases} \sqrt[3]{x}, & x < 1 \\ \sqrt[6]{x}, & x \geq 1 \end{cases}$$



Don't graph the dotted lines! I used them to guide my picture.