Test 1 Solutions (selected)

\# 1.a) $x = .\overline{27}$

$$\begin{aligned} 100x &= 27.2727\ldots \\ - \quad x &= .2727\ldots \\ \hline 99x &= 27 \\ x &= \frac{27}{99} = \frac{3}{11} \end{aligned}$$

c) $\{(1,2),(-3,4),(7,0),(2,4),(5,-1)\}$ has no inverse because it is not one-one. $f(-3) = f(2) = 4$.

d) $f(x) = \frac{x^3}{\sqrt{1-x^2}}$ Choices indicate that odd, even needs checking.

$$f(-x) = \frac{(-x)^3}{\sqrt{1-(-x)^2}} = \frac{-x^3}{\sqrt{1-x^2}} = -f(x)$$ odd fcn.

f) $g(x) = \frac{6x+12}{x}$ The y-intercept is $g(0)$, which does not exist here.

\# 3b) $$\sqrt[3]{250} - \sqrt[3]{54} = \sqrt[3]{125 \cdot 2} - \sqrt[3]{27 \cdot 2} = 5\sqrt[3]{2} - 3\sqrt[3]{2} = 2\sqrt[3]{2}$$

c) $$\frac{\cancel{(x+3)}\cancel{(x-2)}\cancel{(x+4)}(x-1)}{\cancel{(x+4)}\cancel{(x+3)}(x+2)\cancel{(x-2)}} = \frac{x-1}{x+2}$$

$x \neq -4, -3, 2$

Although $x \neq -2$, the important restrictions are those which make the original expression equivalent to the final

So, if you were to graph $y =$ given expression, you'd only need to graph $\boxed{y = \frac{x-1}{x+2}}$

but exclude $\boxed{x = -2, +2, -3, -4}$ from the domain.

#4 a) $x(3x+16) = 35 \rightarrow \cancel{3x^2 + 6x - 35 = 0}$

$3x^2 + 16x - 35 = 0$ factor + solve

$(3x-5)(x+7) = 0$ to get $\boxed{x = \frac{5}{3}, -7}$

$3x - 5 = 0 \quad x + 7 = 0$

b) Group $(x^3 + 3x^2) - (4x + 12) = 0$

factor $x^2(x+3) - 4(x+3) = 0$

factor $(x+3)(x^2-4) = 0$

factor $(x+3)(x+2)(x-2) = 0 \qquad x = -3, \pm 2$

c) $\sqrt{3x+4} - \sqrt{9x} = -2$

$\sqrt{3x+4} = \sqrt{9x} - 2$ ← recommended approach

$3x+4 = 9x - 4\sqrt{9x} + 4$ ← square both sides

$3x + \cancel{4} = 9x - 12\sqrt{x} + \cancel{4}$ ← simplify!

$-6x = -12\sqrt{x}$ ~~all but move radicals to one side~~

$x = 2\sqrt{x}$ ← reduce

$x^2 = 4x$ ← square

$x^2 - 4x = 0$ ← set = zero

$x = \cancel{0}, 4$ check (always! for $\sqrt{\ }$ equations)

Only $x = 4$ works in original

#5

$$
\begin{array}{r}
2x^2 + 7x - 4 \\
x^2 - x - 2 \,\big)\overline{\,2x^4 + 5x^3 - 15x^2 - 10x + 8} \\
-(2x^4 - 2x^3 - 4x^2) \downarrow \\
\hline
7x^3 - 11x^2 - 10x \\
-(7x^3 - 7x^2 - 14x) \\
\hline
-4x^2 + 4x + 8 \\
-(-4x^2 + 4x + 8) \\
\hline
0
\end{array}
$$

#6 The set up to complete the square is:

$$-3(x^2 + 8x + \underset{\uparrow\ (b/2)^2}{16}) - 43 \underset{\uparrow}{+ 48}$$

$-3(b/2)^2$ has to be added back, that is, $+3(16)$

$$\boxed{-3(x+4)^2 + 5 \quad \text{final}}$$

#7. Assume $f(a) = f(b)$ + show that $a = b$ from this assumption

$$\frac{1-5a}{1+2a} = \frac{1-5b}{1+2b}$$

$$(1-5a)(1+2b) = (1-5b)(1+2a)$$

$$\vdots$$

$$-7a = -7b$$

$$a = b \checkmark$$

But see NOTE below

$$f(x) = \frac{1-5x}{1+2x} \qquad f^{-1}(x) = ?$$

Use y for $f(x)$, reverse the variables, and solve for the new y.

$$y = \frac{1-5x}{1+2x} \longrightarrow x = \frac{1-5y}{1+2y}$$

$$\longrightarrow x(1+2y) = 1-5y$$

$$\longrightarrow x + 2xy = 1-5y$$

$$\longrightarrow x - 1 = -2xy - 5y$$

$$\longrightarrow x-1 = y(-2x-5)$$

$$\longrightarrow \frac{x-1}{-2x-5} = y = f^{-1}(x)$$

This simplifies to $\boxed{f^{-1}(x) = \frac{1-x}{2x+5}}$

<u>Note</u>: The reason it is not enough to say there is a unique x ~~for~~ every y + a unique y for every x — <u>which is</u> a valid definition of one-one, — is that given an actual <u>function</u> you'd have to show it for infinite pairs!! Or graph it.

So, we $\underline{\text{must}}$ use the other version of the definition and apply it to the given ~~#8e~~ function, as shown.

~~#8e) $f(-2) = -5$ $f(10) = 64$ $(f \circ h)(4) = 1$~~
~~from $h(4) = 3$~~
~~& $f(3) = (3-2)^2 = 1$~~

f) If f were one - one, then if $f(a) = f(b)$ for $\underline{\text{any}}$ $a, b \in \operatorname{dom} f$, it would be true that $a = b$.

We can find a counterexample easily, namely $f(0) = 3 \cdot 0 + 1 = 1$
$f(3) = (3-2)^2 = 1$

~~See how $f(0) = f(3)$ but $0 \neq 3$?~~

It's even easier if you just look at f on either side of $x = 2$.

~~$f(1.5) = (1.5-2)^2 = .5^2$~~
$f(2.5) = (2.5-2)^2 = .5^2$

So f is $\underline{\text{not}}$ one - one.

\# 8. $f(x) = \begin{cases} 3x+1, & x \le 0 \\ (x-2)^2, & x > 1 \end{cases}$

$g(x) = \dfrac{1}{x^2-4}$ $\qquad h(x) = \sqrt{x+5}$

a) $\text{Dom } f = (-\infty, 0] \cup (1, \infty)$ from given

$\text{Dom } g = (-\infty, -2) \cup (-2, 2) \cup (2, \infty)$

since $x \neq \pm 2$

$\text{Dom } h = [-5, \infty)$ since $x+5 \geq 0$

$x \geq -5$

b) $(g+h)(x) = \dfrac{1}{x^2-4} + \sqrt{x+5}$ } Dom is intersection of Dom f + Dom g

c) $(g \circ h)(x) = g(\sqrt{x+5})$

$$= \frac{1}{(\sqrt{x+5})^2 - 4} = \frac{1}{x+5-4} = \frac{1}{x+1}$$

The domain is found from ~~simplifi~~ the <u>un</u>simplified expression $\dfrac{1}{\sqrt{x+5}^{\,2} - 4}$.

$x \neq -1$, obviously, but from ↗ $x+5 \geq 0$

so the domain of h intersects with $x \neq -1$ to give Dom $g \circ h$.

$$\boxed{\text{Dom } g \circ h = [-5, -1) \cup (-1, \infty)}$$