Math 108: Exam 2 **Name** KEY

Fall 2013 **Instructor** _____

Page	1	2	3	4	5	6		Total	Course Points
Points	18	18	24	20	18	20		118	150
Score									

- Calculators are not permitted for this test.
- Show your work unless the problem requires only a short answer.
- There are problems on both front and back of the pages.
- If you need scrap paper, ask your instructor. You may not use your own. If you do use scrap paper, make sure to hand it in at the end of the exam.

1. (18 points – 3 points each) For each of the following, choose the ONE BEST answer:

Suppose $f(x) = 2x^2 - 4x + c$. For which values of c does the function have two real roots?

(a) $c = 2$ (b) $c < 2$ (c) $c > 4$ (d) $c > -2$ (e) none of these

$b^2 - 4ac$
$16 - 4(2)c > 0$
$16 - 8c > 0$
$16 > 8c$
$2 > c$

Suppose g is a transformation of f such that $g(x) = -f(x) - 2$. Which of these is true?
(a) g represents a reflection of f through the y-axis with a vertical shift down 2 units.
(b) g represents a reflection of f through the x-axis with a vertical shift down 2 units.
(c) g represents a reflection of f through the x-axis with a horizontal shift right 2 units.
(d) g represents a reflection of f through the x-axis with a horizontal shift left 2 units.
(e) none of these

Which of the following is NOT true of $f(x) = \frac{x^2-5x-6}{x+2}$?
(a) Its graph has a horizontal asymptote
(b) (0, -3) is its y-intercept
(c) Its graph has two distinct x-intercepts
(d) $x = -2$ is a vertical asymptote on its graph
(e) All of the above choices are true

The range of the function $f(x) = \sqrt[6]{x}$ is:
(a) all real numbers (b) $[1, \infty)$ (c) $(-\infty, 0]$ (d) $[0, \infty)$ (e) none of these

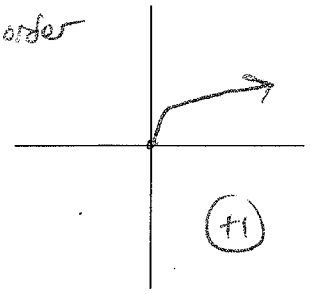
The slope of the line $2x + 3y = 5$ is:
(a) $\frac{2}{3}$ (b) $\frac{3}{2}$ (c) $-\frac{2}{3}$ (d) $\frac{5}{3}$ (e) none of these

Which of the following numbers is not a possible rational root of the polynomial function given by $f(x) = 3x^8 + 5x^7 - 13x^5 + 19x^2 - 11$, according to the Rational Root Theorem?

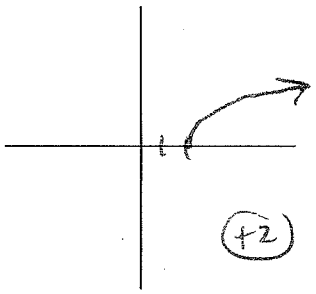
(a) -11 (b) $\frac{1}{3}$ (c) -1 (d) 3 (e) All of these are possible rational roots.

2. (9 pts.) Sketch the graph of $g(x) = 3\sqrt[4]{-x-2}+1$. Do this by starting with the graph of $y = \sqrt[4]{x}$ and applying one transformation at a time until you are finished. Show each step. Label each intermediate graph with its corresponding equation. Scale your graphs sufficiently such that the position of each graph is clear. You might not need all of the axes below.

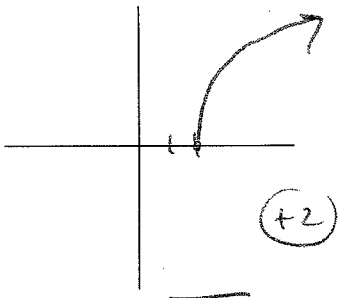
(-3) out of order



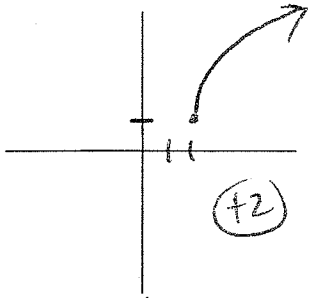
$y = \sqrt[4]{x}$



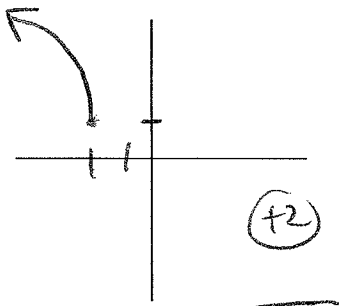
$y = \sqrt[4]{x-2}$



$y = 3\sqrt[4]{x-2}$



$y = 3\sqrt[4]{x-2}+1$



$g(x) = 3\sqrt[4]{-x-2}+1$

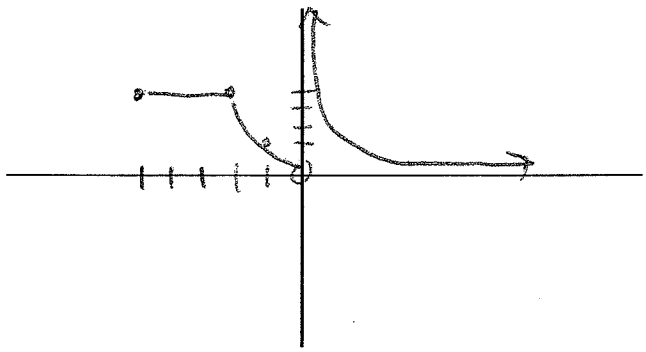
3. (9 pts.) Sketch the graph of the following piecewise-defined function.

$$f(x) = \begin{cases} \frac{1}{x} & if\ x > 0 \\ x^2 & if\ -2 < x < 0 \\ 4 & if\ -5 \le x \le -2 \end{cases}$$

(+3) $\frac{1}{x}$

(+3) x^2 — (+1) (0, 0) hole; (+1) (-2, 4); (+1) shape.

(+3) 4 — (+1) (-2, 4) filled in; (+1) (-5, 4) filled in; (+1) shape



(-3) if not a function.

4. (12 pts.) For this problem suppose $A = (1, -7)$ and $B = (4, 5)$.

(a) Give the equation of the line that goes through points A and B. Write your final answer in slope-intercept form.

4 $m = \frac{5-(-7)}{4-1} = \frac{12}{3} = 4$ (+1)

$y - 5 = 4(x - 4)$ (+2)

$y - 5 = 4x - 16$

$y = 4x - 11$ (+1)

(b) Find the equation of the line that goes through (0, 6) and is perpendicular to the line found in part (a). Any form of the equation of a line is acceptable.

2 $y = -\frac{1}{4}x + 6$

(+1) slope of $-\frac{1}{4}$

(+1) some correct equation

(c) Write the equation of the horizontal line that goes through B.

2 $y = 5$ (+2) all or nothing

(d) Find the midpoint of the line segment joining A and B.

2 $\left(\frac{1+4}{2}, \frac{-7+5}{2}\right) = \left(\frac{5}{2}, -1\right)$

(+1) x-coordinate

(+1) y-coordinate

(e) Determine the length of the line segment joining A and B.

2 $d = \sqrt{(4-1)^2 + (5-(-7))^2} = \sqrt{3^2 + 12^2} = \sqrt{9+144} = \sqrt{153}$

(+2) (-1) arithmetic error

(-2) fundamental error.

5. (12 pts.) Solve each equation for x.

(a) $-3x^2 - 7x + 5 = 0$

$a = -3 \quad b = -7 \quad c = 5$

$x = \frac{-(-7) \pm \sqrt{(-7)^2 - 4(-3)(5)}}{2(-3)} = \frac{7 \pm \sqrt{109}}{-6}$

(+2) (+2)

(b) $x^4 - 7x^2 = -12$

$x^4 - 7x^2 + 12 = 0$ (+1)

$(x^2 - 4)(x^2 - 3) = 0$ (+1)

$x = \pm 2$ (+1) $x = \pm\sqrt{3}$ (+1)

(c) $(12x + 5)^2 = 11$

$12x + 5 = \pm\sqrt{11}$ (+2)

$12x = -5 \pm \sqrt{11}$ (+1)

$x = \frac{-5 \pm \sqrt{11}}{12}$ (+1)

6. (20 points)

Two of the roots of function $g(x) = -x^4 + 5x^3 - 2x^2 - 20x + 24$ are $x = 2$ and $x = -2$.

all or nothing 1 (a) What is the degree of g? 4

all or nothing 1 (b) What is the leading coefficient of g? -1

all or nothing 2 (c) Use arrows to describe the "end behavior" of the graph of g. ↙ ---- ↘

all or nothing 1 (d) Find the y-intercept of g. $(0, 24)$

6 (e) Write the COMPLETE factorization for g.

$$x^2 - 4 \overline{)-x^4 + 5x^3 - 2x^2 - 20x + 24}$$

$-x^2 + 5x - 6$

$-(-x^4 \quad +4x^2)$

$5x^3 - 6x^2 - 20x$

$-(5x^3 \quad -20x)$

$-6x^2 \quad +24$

$-(-6x^2 \quad +24)$

0

$g(x) = (x+2)(x-2)(-x^2+5x-6)$

$= -(x+2)(x-2)(x^2-5x+6)$

$= -(x+2)(x-2)(x-3)(x-2)$

$= -(x+2)(x-3)(x-2)^2$

(+2) recognition of $x+2$ and $x-2$ as factors

(+2) Divides

(+2) Factors trinomial leftover

(+2 each) 3 (f) Find all x-intercepts of g.

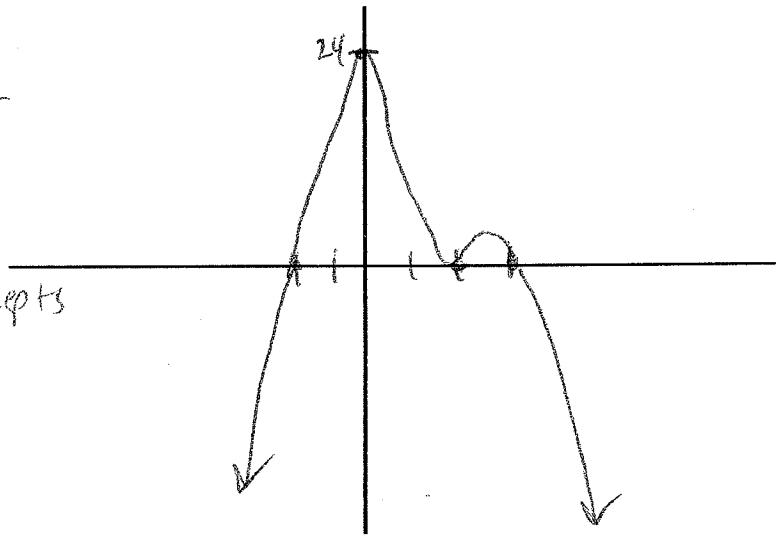
$(-2, 0)$ $(2, 0)$ $(3, 0)$

6 (g) Use the above information to sketch a graph of g on the axes provided. Be sure to consider the multiplicities of the x-intercepts when sketching your graph. Scale the axes as needed.

(+2) end behavior

(+2) intercepts

(+2) behavior at x-intercepts



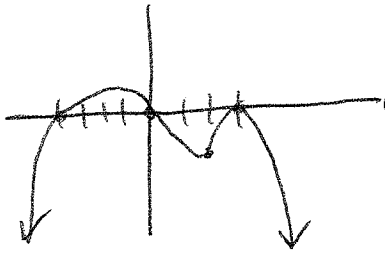
$x^2 + x - 12$

7. (8 points)
Find a polynomial function of <u>least</u> possible degree satisfying ALL of the following conditions.

(a) The polynomial must have $x = 3$, $x = 0$, and $x = -4$ as its only roots.
(b) As $x \to \infty$, $f(x) \to -\infty$
(c) As $x \to -\infty$, $f(x) \to -\infty$
(d) $f(2) < 0$

Express your answer as a function. You may leave your answer in factored form. (-1) if not least possible degree

$f(x) = -x(x-3)^2(x+4)$



(a) (+3) Has x, $x-3$, $x+4$ as factors
(b)(c) (+3) Satisfies (b) and (c) (has even degree and negative LC)
(d) (+2) has correct factor squared $(x-3)^2$

8. (10 pts.) List the <u>equations</u> of all asymptotes for each of the following rational functions. If a function has no asymptotes of a certain type, write "None."

(a) $f(x) = \frac{2x^2+x-5}{x^2}$

Vertical? $x = 0$ (+2)

Horizontal? $y = 2$ (+2)

Slant? None

(b) $g(x) = \frac{x-3}{x^2-x-6} = \frac{x-3}{(x-3)(x+2)} = \frac{1}{x+2}$, if $x \neq 3$.

Vertical? $x = -2$ (+2) (+2) for NOT including $x = 3$ as V.A.

Horizontal? $y = 0$ (+2)

Slant? None

Do not give credit if student lists both a HA and SA.

9. (20 pts.) $f(x) = \frac{x^3-5x^2-6x}{x^2-x-2} = \frac{x(x^2-5x-6)}{(x-2)(x+1)} = \frac{x(x-6)(x+1)}{(x-2)(x+1)}$

(a) Find the domain of f.

(+2) $\{x \mid x \neq 2, -1\}$ (+1) each excluded value

(b) Find the y-intercept of f.

(+2) $f(0) = \frac{0(0-6)}{0-2} = 0$ $(0,0)$ (+1) plugs in 0 (+1) answer

(c) Find all x-intercepts of f.

(+2) $x(x-6) = 0 \Rightarrow x = 0, \ x = 6$ $(0,0)$ (+1) $(6,0)$ (+1)

(d) Find the coordinates of any "holes" in the graph of f.

(+3) $(-1, -\frac{7}{3})$ (+2) $x = -1$ (+1) $y = -\frac{7}{3}$ $\frac{-1(-1-6)}{-1-2} = \frac{7}{-3}$

(e) Find the **equations** of all asymptotes. Identify them as vertical, horizontal, or slant.

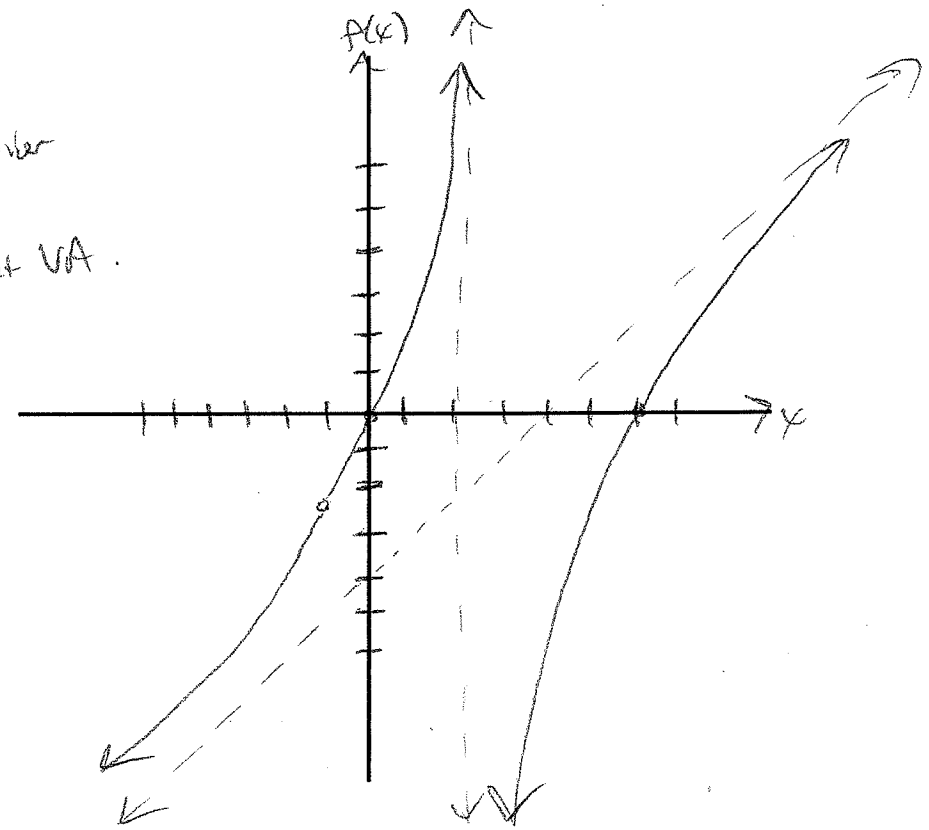
(+2) $\rightarrow x = 2$ (VERTICAL)

(+3) $\rightarrow y = x - 4$ (SLANT)

$$x-2 \overline{)\, x^2 - 6x} \quad x - 4$$
$$-(x^2 - 2x)$$
$$-4x$$

6 (f) Use the above information to sketch the graph of f on the axes provided. Label the axes appropriately.

(+1) hole
(+2) end behavior
(+1) intercepts
(+2) behavior at VA.



(-3) for not a function