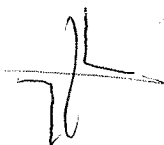




Compare to $y = \frac{1}{x}$

Compare to $y = \frac{1}{x^3}$



In Exercises 5-12, find the domain of the function and identify any horizontal and vertical asymptotes.

1. $f(x) = \frac{1}{x^2}$

6. $f(x) = \frac{4}{(x-2)^3}$

7. $f(x) = \frac{2+x}{2-x}$

8. $f(x) = \frac{1-5x}{1+2x}$

8. $x \neq -1/2 \rightarrow VA: x = -1/2$

$m = n \rightarrow HA: y = 5/2$

$f(0) = 1, f(x) = 0$ at $x = 1/5$

$f(1) = -4/3$

$f(-1) = -6$

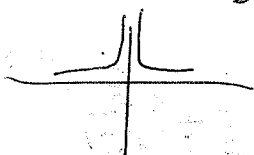


5. $x \neq 0 \rightarrow VA: x = 0$

$m > n \rightarrow HA: y = 0$ (behavior at ends)

$f(x) = f(-x) \rightarrow$ even

$f(0)$ undefined (xy-int), $f(1) = 1, f(-1) = 1$



6. $x \neq 2 \rightarrow VA: x = 2$

$m > n \rightarrow HA: y = 0$

$f(0) = -1/2$

$f(x) \neq 0$ - no roots



7. $x \neq 2 \rightarrow VA: x = 2$

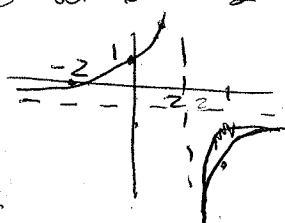
$n = m \rightarrow HA = a_n/b_m = -1$

$f(0) = 1$

$f(x) = 0$ at $x = -2$

$f(1) = 3$

$f(3) = -5$



$y = \frac{2+x}{2-x}$

9. $f(x) = \frac{x^3}{x^2-1}$ odd

10. $f(x) = \frac{2x^2}{x+1}$

11. $f(x) = \frac{3x^2+1}{x^2+x+9}$

12. $f(x) = \frac{3x^2+x-5}{x^2+1}$

9. $x \neq \pm 1 \rightarrow VA: x = 1, x = -1$

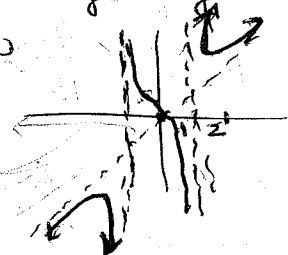
$m > n$ by 1 $\rightarrow$ SA at $y = x$ by long div

$f(0) = 0, f(x) = 0$ at $x = 0$

$f(1/2) = 1/8 / -3/4 = -1/6$

$f(-1/2) = -1/8 / -3/4 = 1/6$

$f(2) = 8/3, f(-2) = -8/3$



$x^2-1 \overline{) x^3 + 0x^2 - x}$
 $-(x^3 - x)$
 $\hline 2x - 2$

$f(x) = x + \frac{x}{x^2-1}$
 $y = x$

$2x - 2 + \frac{x}{x+1}$

$\frac{2x}{x+1} - \frac{2}{x+1}$

$x+1 \overline{) 2x^2}$
 $2x^2 + 2x$

$-2x - 2$

$\frac{-2}{x+1}$

10. $x \neq -1 \rightarrow VA: x = -1$

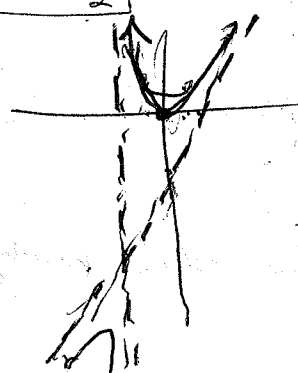
$m > n$ by 1 $\rightarrow$ SA at $y = 2x - 2$ by div

$f(0) = 0$ (x+y-int)

$f(-2) = -8, f(1) = 1$

$f(-1/2) = 1/2 / 1/2 = 1$

$f(1/2) = 1/2 / 3/2 = 1/3$

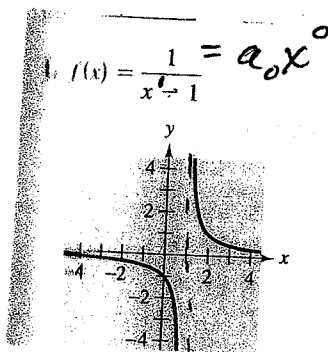


Rational Function Essentials

(Exercises)

$$f(0) = 0$$

$$0 = \frac{5x}{x-1}; x=0$$



$$\text{Dom } x \neq 1$$

$$f(0) = -1$$

$$\frac{1}{x-1} \stackrel{?}{=} 0$$

$$1 \neq 0$$

$$\text{HA: } y = 0$$

$$\text{VA: } x = 1$$

$$n = 0$$

$$m = 1$$

$$2. f(x) = \frac{5x}{1x-1}$$

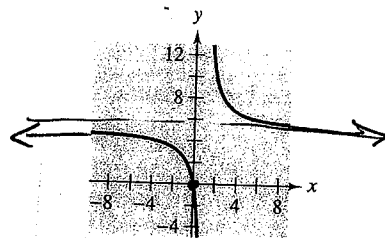
$$n = 1$$

$$m = 1$$

Dom:

$$x \neq 1$$

$$\text{VA: } x = 1$$



$$\begin{array}{r} 5x \\ (x-1) \overline{) 5x} \\ \underline{-(5x-5)} \\ 5 \end{array}$$

$$5 + \frac{5}{x-1} \rightarrow 5 + 0 = 5$$

x gets big

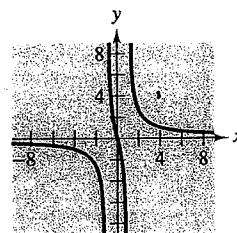
H.A.

$$\boxed{\text{HA} = \frac{a_n}{b_m} \text{ when } n=m}$$

$$4. f(x) = \frac{4x}{x^2-1}$$

$$n < m$$

$$\text{HA: } y = 0$$



$$\frac{4x}{x^2-1} = 0$$

$$x = 0 \text{ root}$$

Inspected x on either side of excluded value & saw it going to large + small values

$$f(x) = \frac{4x}{x^2-1}$$

$$f(x) = f(-x) \text{ odd?}$$

$$-f(x) = f(-x)$$

even? - reflection over y -axis symmetry?

Show it's odd using def.