Practice for trig test

Please note that I altered #9a so it has a horizontal compression.

#1 <u>Coterminal angles</u>: Add or subtract 2π to given θ.

$\theta = \frac{\pi}{3}, \quad \frac{\pi}{3} + 2\pi, \quad \frac{\pi}{3} - 2\pi \quad \boxed{\frac{7\pi}{3}, \frac{-5\pi}{3}}$

Also, $\theta + 2n\pi$, so $\frac{\pi}{3} + 4\pi = \boxed{\frac{13\pi}{3}}$

#2 $s = \theta r$, θ in radians

$s = \left(15\cancel{0} \cdot \frac{\pi}{18\cancel{0}}\right) \cdot 2\text{ft} = \left(\frac{5}{6}\pi\right)(2)$

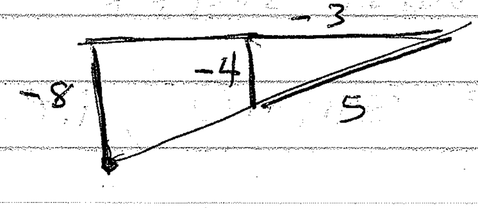
$= \boxed{\frac{10\pi}{6}\text{ ft}}$

#4 $2^2 + (-5)^2 = 29$

$\Rightarrow \left(\frac{2}{\sqrt{29}}, \frac{-5}{\sqrt{29}}\right)$ is pt on unit circle

#5 a) $\cos\alpha = -3/5$ pt $(x, -8)$

Draw the similar triangles in Q3.

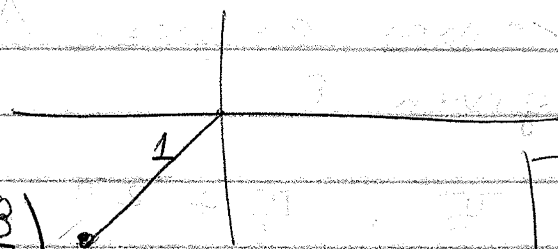


$\frac{-3}{x} = \frac{-4}{-8}, \quad \boxed{x = -6}$

b) Divide $(-6, -8)$ by r of the triangle:

$$6^2 + 8^2 = 100 = r^2$$

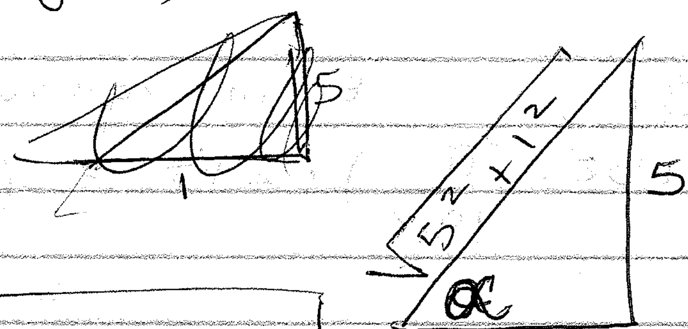
$$r = 10$$



$\left(\frac{-6}{10}, \frac{-8}{10}\right)$

$$\boxed{\therefore \left(-\frac{3}{5}, -\frac{4}{5}\right)}$$

#3 (forgot!) $\tan \alpha = 5$ (that is, $5/1$)



hyp $= \sqrt{26}$

$\sin\alpha = \frac{5}{\sqrt{26}}$ $\quad \csc\alpha = \frac{\sqrt{26}}{5}$ $\quad \cot\alpha = \frac{1}{5}$

$\cos\alpha = \frac{1}{\sqrt{26}}$ $\quad \sec\alpha = \sqrt{26}$

#6 $\sin^2 10^\circ + \sin^2 80^\circ = (\sin 10^\circ)^2 + (\sin 80^\circ)^2$

Think of a relationship btwn 10 + 80, namely $10 + 80 = 90$, which is the cos $\rightarrow$ sin phase shift:

$\cos(90 - 10) = \sin 10$, i.e. $\cos 80 = \sin 10$

So ~~$\sin^2 10 + \cos$~~ $\cos^2 80 + \sin^2 80 = 1$

$(\text{Alternately, } \sin^2 10 + \cos^2 10 = 1)$

#7) a)

S	A
T	C

$\tan\theta = 7 > 0$

$\cos\theta < 0$

$\boxed{Q3}$

b) $\sin\theta > 0$

$\sec\theta = \dfrac{1}{\cos\theta} = 4 > 0$ $\boxed{Q1}$

#8a) $\cos\left(\frac{\pi}{2} - x\right) = \cos\left(x - \frac{\pi}{2}\right)$ is true

because cos is even, so

$\cos(-\theta) = \cos(\theta)$

Thus: $\cos\left(\frac{\pi}{2} - x\right) = \cos\left(-\left(\frac{\pi}{2} - x\right)\right)$

$= \cos\left(x - \frac{\pi}{2}\right)$

b) $\sin\left(\frac{\pi}{2} - x\right) \neq \sin\left(x - \frac{\pi}{2}\right)$

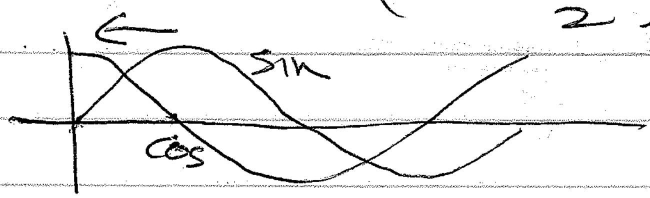
since $\sin\theta$ is odd,

thus $\sin(-\theta) = -\sin\theta$

so $\sin\left(-\left(\frac{\pi}{2} - x\right)\right) = -\sin\left(x - \frac{\pi}{2}\right)$

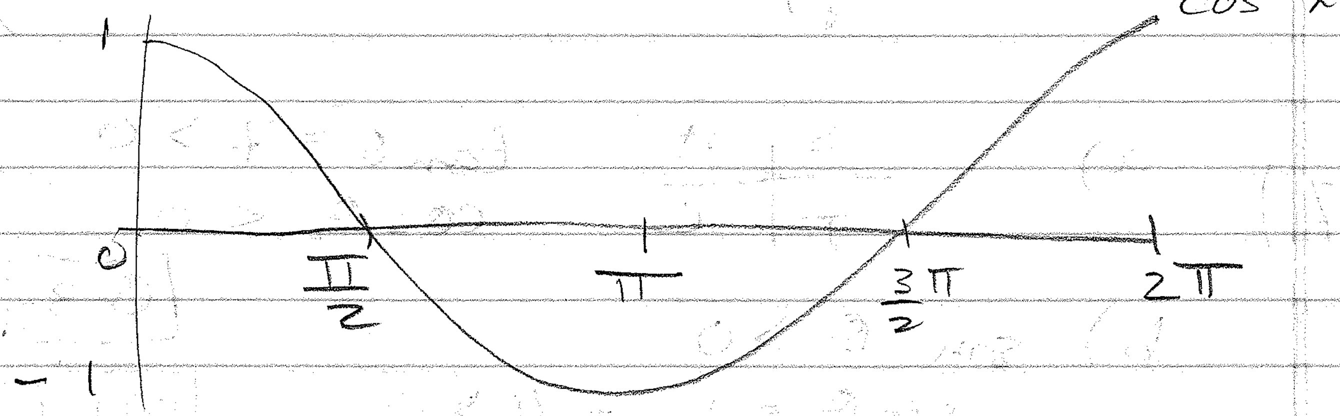
$\neq \sin\left(x - \frac{\pi}{2}\right)$

c) $\cos x = \sin\left(x + \frac{\pi}{2}\right)$ is true

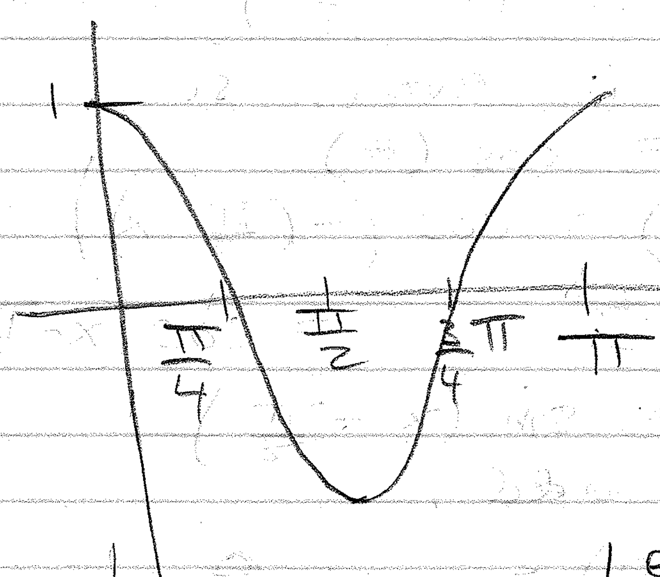


#9a) with a change in B:

$$y = \cos\left(2x + \frac{\pi}{3}\right) + 1$$



$B = 2$ so $Per = \frac{2\pi}{2} = \pi$

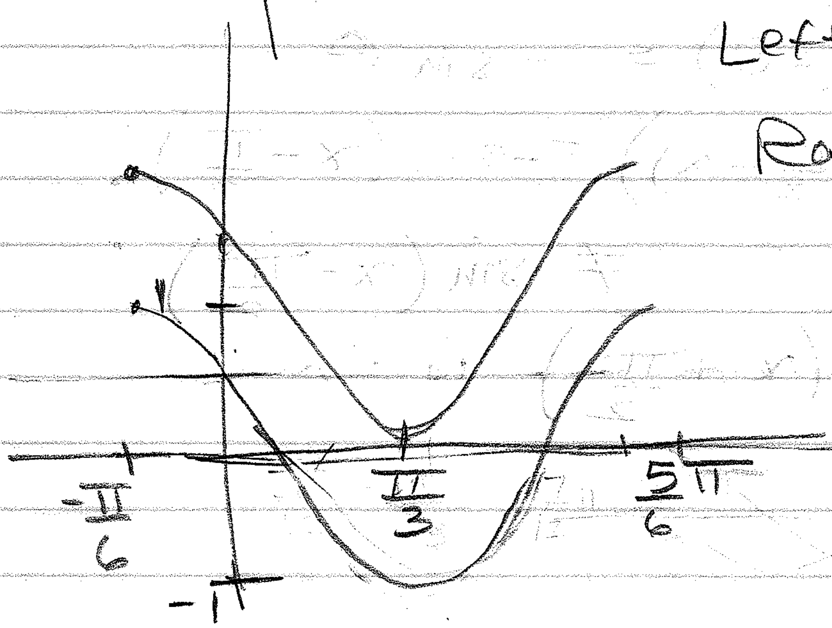


All the x values are halved.

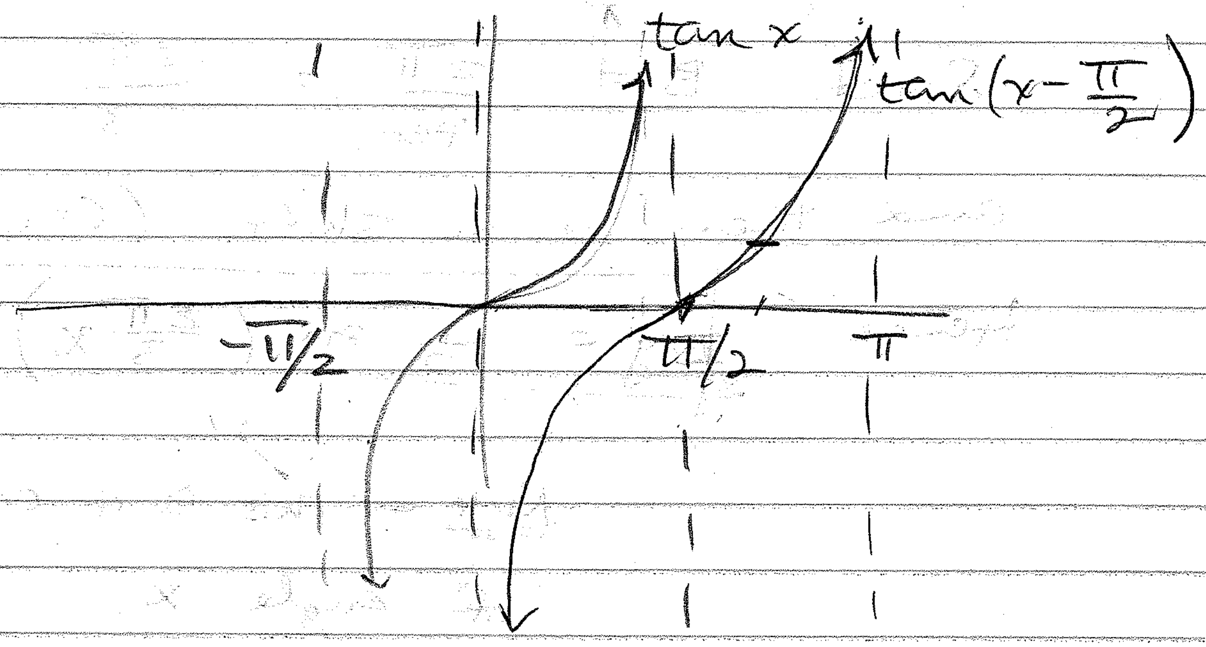
Left shift $= \frac{C}{B} = \frac{\pi/3}{2} = \frac{\pi}{6}$

Raise up $D = 1$

Notice the minimum is at $\frac{\pi}{2} - \frac{\pi}{6} = \frac{2\pi}{6} = \frac{\pi}{3}$



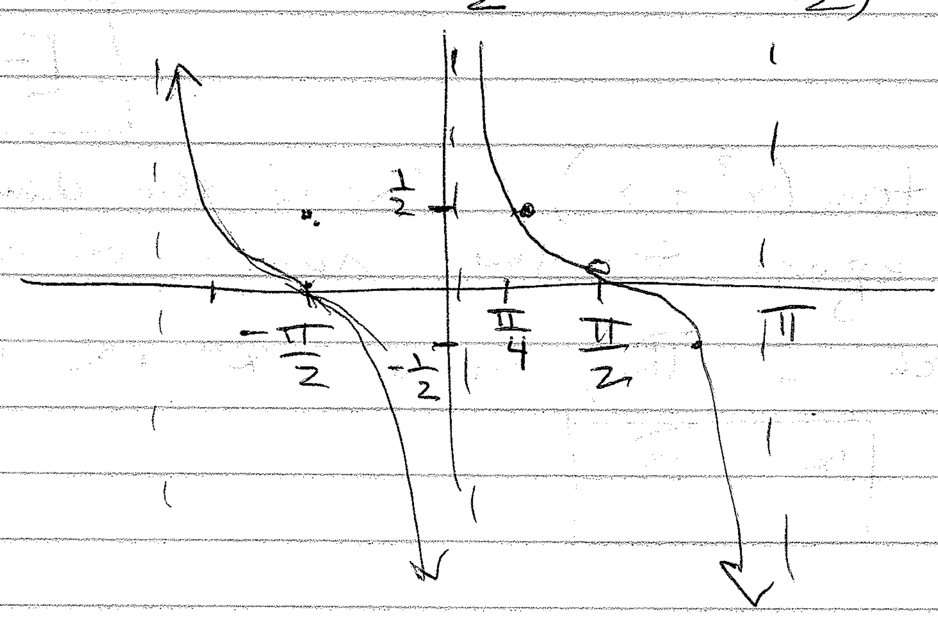
#9b $g(x) = -\frac{1}{2}\tan\left(x-\frac{\pi}{2}\right)$



- Since $\tan\left(\frac{\pi}{4}\right) = 1$, then $\frac{1}{2}\tan\left(\frac{\pi}{4}\right) = \frac{1}{2}$

But we shifted $\frac{\pi}{2}$ right, so
so $\tan\theta = \frac{1}{2}$ at $x = 3\pi/4$.

- Then we flip the graph (over x-axis) since we have $-\frac{1}{2}\tan\left(x-\frac{\pi}{2}\right)$.



#10) $y = \frac{1}{2}\sin(Bx + C) - 1$

$A = \frac{1}{2}$ + $D = -1$ were simple.

Since $B = \frac{2\pi}{Per} = \frac{2\pi}{3}$

and there is no shift $(C = 0)$

Hence $\boxed{y = \frac{1}{2}\sin\left(\frac{2\pi}{3}x\right) - 1}$

Note – This is a coeff of angle x.

#11) a) $f(x) = \cos^{-1}(x+3)$

Dom f are those x such that

$-1 \leq x + 3 \leq 1 \Longrightarrow -4 \leq x \leq -2$

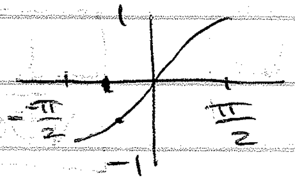
$\boxed{[-4, -2]}$

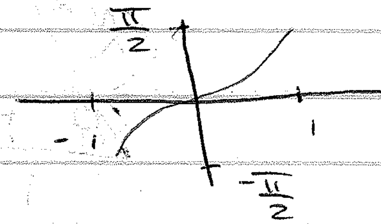
b) $\arctan(x^2 + x)$. Since the dom of $\tan^{-1}$ is equal to the range of the tan, which is $\mathbb{R}$, then $x^2 + x \in \mathbb{R}$,

so $\boxed{x \in \mathbb{R}}$

#12) a) $\sin^{-1}(-\sqrt{2}/2) = \boxed{-\pi/4}$ on $[-\frac{\pi}{2}, \frac{\pi}{2}]$

Stop here + ask why we chose $-\pi/4$ rather than $\frac{5\pi}{4}$ or $\frac{7\pi}{4}$. It's b/c the ~~[illegible]~~ sine fcns' restricted domain is $[-\frac{\pi}{2}, \frac{\pi}{2}]$, so the range of arc sin x must be in this interval.





b) $\cos^{-1}(\cos(5\pi/4))$ on $[0, \pi]$ which is the restricted domain of the cosine fcn. in order for an inverse to be defined. Find the ref for $\frac{5\pi}{4}$ in $[0, \pi]$.

$$\cos^{-1}(\cos\underbrace{(5\pi/4)}_{\text{negative}}) = \cos^{-1}\left(\cos\left(\frac{3\pi}{4}\right)\right)$$

$$= \boxed{\frac{3\pi}{4}}$$

c) $\tan^{-1}(\tan \pi/6) = \pi/6$, which is in $(-\pi/2, \pi/2)$ the restricted dom.

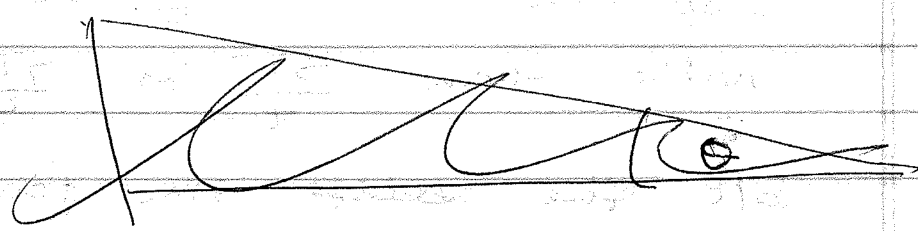
d) $\sec\left(\cos^{-1}\left(\frac{1}{2}\right) - \sin^{-1}\left(\frac{\sqrt{3}}{2}\right)\right)$

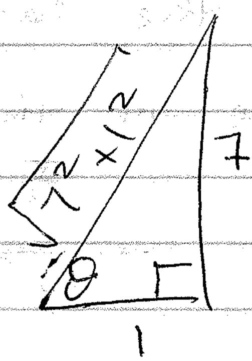
$$= \sec\left(\frac{\pi}{3} - \frac{\pi}{6}\right) = \sec\left(\frac{\pi}{6}\right) = \frac{1}{\cos\frac{\pi}{6}} \boxed{= \frac{2}{\sqrt{3}}}$$

e) $\cos\left(\underbrace{\tan^{-1}(7)}\right)$

Draw the triangle with θ that yields $\tan\theta = 7/1$

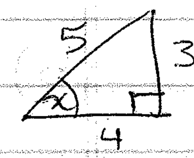
Nope



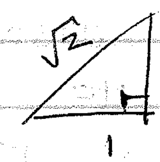


$\cos\theta$ is seen here to be $\boxed{\dfrac{1}{\sqrt{50}}} = \boxed{\dfrac{1}{5\sqrt{2}}}$

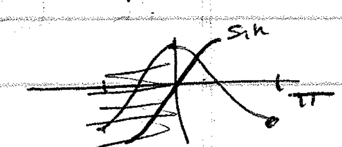
13) a) $\sin^{-1}\left(\frac{3}{5}\right) = \cos^{-1}\left(\frac{4}{5}\right)$



b) $\tan^{-1} 1 = \sin^{-1}(\sqrt{2}/2)$



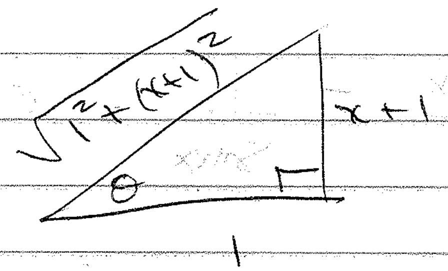
c) $\underbrace{\cos^{-1}(-1)} = \sin^{-1}(x)$



Choose this, since you need to be in $[0, \pi]$

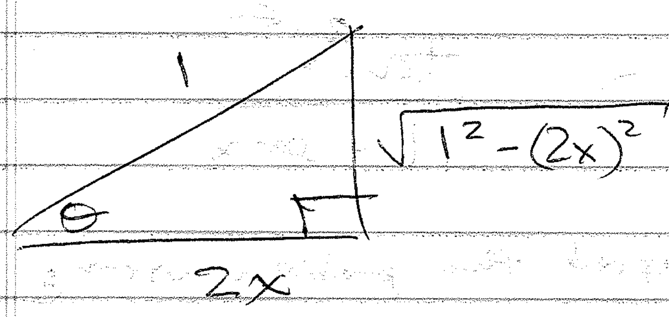
Since $\sin^{-1}(x)$ is not defined in $[0, \pi]$, but in $[-\pi/2, \pi/2]$, there is $\boxed{\text{no solution.}}$ (Note – If I gave $\sin^{-1}(0)$ in class, that's wrong. The point of this is to check domains.)

#14 $\cos\left(\tan^{-1}(x+1)\right)$ Draw the triangle with $\tan\theta = \frac{x+1}{1}$

a) 

$$\Rightarrow \boxed{\cos\theta = \frac{1}{\sqrt{1+(x+1)^2}}}$$

b) $\sin\left(\arccos(2x)\right)$ Draw the triangle with $\cos\theta = \frac{2x}{1}$



$$\Rightarrow \boxed{\sin\theta = \sqrt{1-4x^2}}$$

#15 Verify identities:

$$\frac{\sin x}{1+\cos x} + \cot x \stackrel{?}{=} \csc x$$

$$\frac{\sin x}{1+\cos x} + \frac{\cos x}{\sin x} \stackrel{?}{=} \csc x$$

$$\frac{\sin x}{1+\cos x} \cdot \frac{1-\cos x}{1-\cos x} + \frac{\cos x}{\sin x}$$

Introduces new dom restriction? $\cos x \neq 1$

$$= \frac{\sin x\,(1-\cos x)}{1-\cos^2 x} + \frac{\cos x}{\sin x}$$

$$= \frac{\cancel{\sin x}(1-\cos x)}{\sin^2 x} + \frac{\cos x}{\sin x}$$

$$= \frac{1-\cancel{\cos x}+\cancel{\cos x}}{\sin x} = \frac{1}{\sin x} = \csc x \checkmark$$

where $\sin x \neq 0$, hence where $\cos x \neq 1$, so we didn't add a new restriction on dom.

b) $$\frac{1-\cos x}{\cos x} = \frac{\tan^2 x}{1+\sec x}$$

~~I~~ I copied this problem wrong. See text # 3m I left out the square.

Working on the right:

$$\cancel{\frac{\tan^2 x}{1+\sec x} = \frac{1+\sec^2 x}{(1+\sec^2 x)}}$$

$$\frac{\tan^2 x}{1+\sec x} = \frac{\sec^2 x - 1}{1+\sec x} = \frac{(\sec x-1)\cancel{(\sec x+1)}}{\cancel{(1+\sec x)}}$$

$$= \sec x - 1 = \frac{1}{\cos x} - 1 = \frac{\cos x - 1}{\cos x}$$

$\checkmark$

#16 a) $\cos^2\left(\frac{\pi}{8}\right) - \sin^2\left(\frac{\pi}{8}\right) = \cos\left(2 \cdot \frac{\pi}{8}\right) = \boxed{\cos\frac{\pi}{4} = \frac{\sqrt{2}}{2}}$

from double $\measuredangle$ formula.

b) This is # 4d in Sec 8.8. I gave the answer to the question!! Namely

$$\frac{\tan\left(\frac{\pi}{5}\right) - \tan\left(\frac{\pi}{30}\right)}{1 - \tan\left(\frac{\pi}{5}\right)\tan\left(\frac{\pi}{30}\right)}$$

which you should recognize as $\tan\left(\frac{\pi}{5} - \frac{\pi}{30}\right)$

$$= \tan\left(\frac{6\pi - \pi}{30}\right) = \tan\left(\frac{5\pi}{30}\right) = \tan\left(\frac{\pi}{6}\right)$$

$$= \frac{\sin \pi/6}{\cos \pi/6} = \frac{1/2}{\sqrt{3}/2} = \boxed{\frac{1}{\sqrt{3}} = \frac{\sqrt{3}}{3}}$$

c) $\sin\left(\frac{5}{12}\pi\right) = \sin\left(\frac{3}{12}\pi + \frac{2}{12}\pi\right)$

$$= \sin\left(\frac{\pi}{4} + \frac{\pi}{6}\right) = \sin\frac{\pi}{4}\cos\frac{\pi}{6} + \cos\frac{\pi}{4}\sin\frac{\pi}{6}$$

$$= \frac{\sqrt{2}}{2}\,\frac{\sqrt{3}}{2} + \frac{\sqrt{2}}{2}\,\frac{1}{2}$$

$$= \frac{\sqrt{6}}{4} + \frac{\sqrt{2}}{4} = \boxed{\frac{\sqrt{6}+\sqrt{2}}{4}}$$

d) $\cos\left(22\frac{1}{2}^\circ\right) = \cos\left(\frac{1}{2} \cdot 45\right)$

$$= \pm\sqrt{\frac{1+\cos 45}{2}} = \pm\sqrt{\frac{1+\sqrt{2}/2}{2}} =$$

$$\pm\sqrt{\frac{\frac{1+\sqrt{2}}{2}}{2}} = \pm\sqrt{\frac{1+\sqrt{2}}{4}} = \frac{\pm\sqrt{1+\sqrt{2}}}{2}$$

#18) a) $\sin ? = \pm\sqrt{\frac{1-\cos 40}{2}}$

$$? = \frac{40}{2} = 20$$

So $\sin 20 = \pm\sqrt{\frac{1-\cos 40}{2}}$

*b) $\cos(4x) = \pm\sqrt{\frac{1+\cos 8x}{2}}$

* miscopied the operation on the master as subtraction — I'm sorry.

#19) $\sin^4 x = (\sin^2 x)^2 = \left(\frac{1-\cos(2x)}{2}\right)^2$

$$= \frac{1-\cos(2x)}{2} \cdot \frac{1-\cos(2x)}{2}$$

$$= \frac{1-2\cos(2x)+\cos^2(2x)}{\cancel{8}\ 4}$$

$$= \frac{1-2\cos(2x)+\frac{1+\cos(4x)}{2}}{\cancel{8}\ 4}$$

$$= \frac{2-4\cos(2x)+1+\cos(4x)}{8} = \boxed{\frac{3-4\cos(2x)+\cos(4x)}{8}}$$

#9) a) $\sin\left(\frac{\pi}{8}\right) = \sin\left(\frac{1}{2} \cdot \frac{\pi}{4}\right)$

$$= \pm\sqrt{\frac{1-\cos(\pi/4)}{2}}$$

$$= \pm\sqrt{\frac{1-\sqrt{2}/2}{2}} = \pm\sqrt{\frac{2-\sqrt{2}}{4}}$$

$$= \boxed{\frac{\pm\sqrt{2-\sqrt{2}}}{2}}$$

b) $\cos(195°) = \cos(45° + 150°)$

$$= \cos(45)\cos(150) + \sin(45)\sin(150)$$

~~$= (\cos^2 48 - \sin^2$~~

270
195
75

$$= -\frac{\sqrt{2}}{2}\cos(30) + \frac{\sqrt{2}}{2}\sin(30)$$

$$= -\frac{\sqrt{2}}{2}\frac{\sqrt{3}}{2} + \frac{\sqrt{2}}{2}\frac{1}{2}$$

$$= \boxed{\frac{\sqrt{2}-\sqrt{6}}{4}}$$

#20) See our class notes

a) $-\cot(2x) = 2 + \tan(2x)$

$-\dfrac{1}{\tan(2x)} = 2 + \tan(2x)$ LCD $\tan(2x)$

$-1 = 2\tan(2x) + \tan^2(2x)$

$\tan^2(2x) + 2\tan(2x) + 1 = 0$

$(\tan(2x) + 1)^2 = 0$

$\tan(2x) = -1$

when $2x = \dfrac{5\pi}{4}$ on $[0, 2\pi)$

$$\boxed{x = \frac{5\pi}{8}}$$

b) $2\sin^2 x + \sin x - 1 = 0$

$(2\sin x - 1)(\sin x + 1) = 0$

$2\sin x = 1$	$\sin x = -1$
$\sin x = \frac{1}{2}$	$x = \frac{3}{2}\pi$
$x = \frac{\pi}{6}, \frac{5\pi}{6}$	