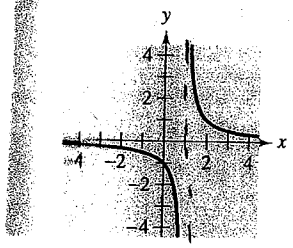
Rational Function Essentials

(Exercises)

$f(0) = 0$

$0 = \frac{5x}{x-1}$; $x = 0$

1. $f(x) = \frac{1}{x-1} = a_0 x^0$



$n = 0$

$m = 1$

Dom $x \neq 1$

$f(0) = -1$

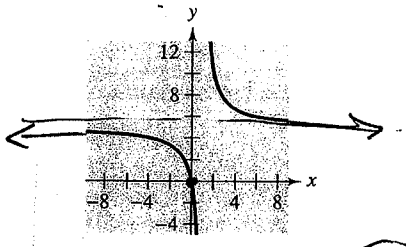
$\frac{1}{x-1} \stackrel{?}{=} 0$

$1 \neq 0$

HA: $y = 0$

VA: $x = 1$

2. $f(x) = \frac{5x}{x-1}$ $n = 1$ $m = 1$ Dom: $x \neq 1$



VA: $x = 1$

$\frac{5x}{x-1}$ $\quad 5 + \frac{5}{x-1}$ → 0

$x-1 \overline{)\,5x}$

$-(5x-5)$

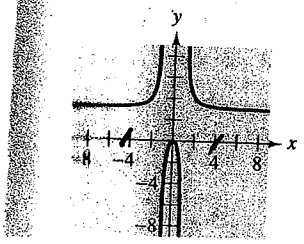
5

$5 + \frac{5}{x-1} \longrightarrow 5 + 0 = 5$

x gets big H.A.

$HA = \frac{a_n}{b_m}$ when $n = m$

3. $f(x) = \frac{3x^2}{x^2-1} = 0$



$f(0) = 0$, roots, $x = 0$

Dom: $x \neq +1, -1$

$n = m$

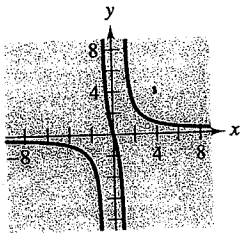
HA: $3/1 = 3 = y$

VA: $x = 1$

$x = -1$

$f(4) = \frac{48}{15} > 3$

4. $f(x) = \frac{4x}{x^2-1}$



$n < m$

HA: $y = 0$

$\frac{4x}{x^2-1} = 0$

$x = 0$ root

Inspected x on either side of excluded value & saw it going to large + small values

$f(x) = \frac{4x}{x^2-1}$

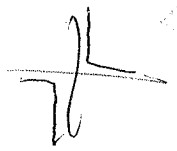
$f(x) = f(-x)$ odd?

$-f(x) = f(-x)$ even? – reflection over y-axis

symmetry?

Show it's odd using def.

Compare to $y = \frac{1}{x}$ Compare to $y = \frac{1}{x^3}$



In Exercises 5–12, find the domain of the function and identify any horizontal and vertical asymptotes.

5. $f(x) = \frac{1}{x^2}$ 6. $f(x) = \frac{4}{(x-2)^3}$

7. $f(x) = \frac{2+x}{2-x}$ 8. $f(x) = \frac{1-5x}{1+2x}$

8. $x \neq -1/2 \Rightarrow$ VA: $x = -1/2$

$n = m \rightarrow$ HA: $y = -5/2$

$f(0) = 1$, $f(x) = 0$ at $x = 1/5$

$f(1) = -4/3$

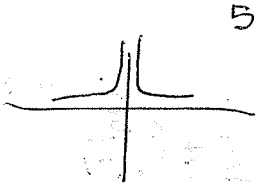
$f(-1) = -6$

5. $x \neq 0 \rightarrow$ VA: $x = 0$

$n > n \rightarrow$ HA: $y = 0$ (behavior at ends)

$f(x) = f(-x) \rightarrow$ even

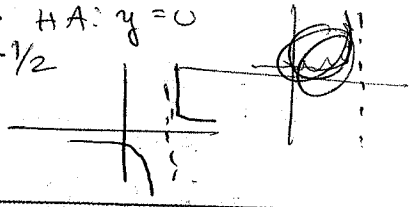
$f(0)$ undefined. (y-int), $f(1) = 1$, $f(-1) = 1$



6. $x \neq 2 \rightarrow$ VA $x = 2$ $f(x) \neq 0$ ~~no roots~~

$n > n \rightarrow$ HA: $y = 0$

$f(0) = -1/2$



7. $x \neq 2 \rightarrow$ VA $x = 2$

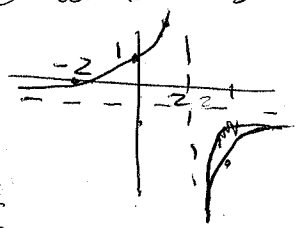
$n = m \rightarrow$ HA $= a_n/b_m = -1$

$f(0) = 1$

$f(x) = 0$ at $x = -2$

$f(1) = 3$

$f(3) = -5$



$y = \frac{2+x}{2-x}$

9. $f(x) = \frac{x^3}{x^2-1}$ odd 10. $f(x) = \frac{2x^2}{x+1}$

11. $f(x) = \frac{3x^2+1}{x^2+x+9}$ 12. $f(x) = \frac{3x^2+x-5}{x^2+1}$

9. $x \neq \pm 1 \rightarrow$ VA: $x = 1$, $x = -1$

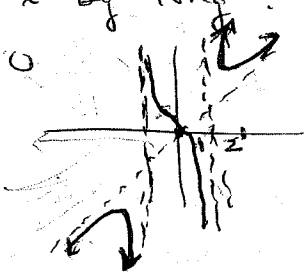
$m > n$ by 1 $\rightarrow$ SA at $y = x$ by long ÷

$f(0) = 0$ $f(x) = 0$ at $x = 0$

$f(1/2) = 1/8 / -3/4 = -1/6$

$f(-1/2) = -1/8 / -3/4 = 1/6$

$f(2) = 8/3$ $f(-2) = -8/3$



$$x^2 - 1 \overline{)\, x^3 + 0x} \quad \frac{x}{x^2-1}$$

$$-(x^3 - x)$$

$$f(x) = \underbrace{x}_{y=x} + \frac{x}{x^2-1}$$

10. $x \neq -1 \rightarrow$ VA: $x = -1$

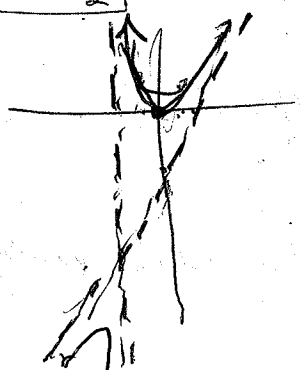
$m > n$ by 1 $\rightarrow$ SA at $y = 2x - 2$ by ÷

$f(0) = 0$ (x+y-int)

$f(-2) = -8$ $f(1) = 1$

$f(-1/2) = 1/2 / 1/2 = 1$

$f(1/2) = 1/2 / 3/2 = 1/3$



$$x + 1 \overline{)\, 2x^2} \quad 2x - 2 + \frac{2}{x+1}$$

$$2x^2 + 2x$$

$$-2x$$

$$-2x - 2$$

$$2$$

11. $f(x) = \frac{3x^2+1}{x^3+x+9}$ $n = m$ $f(0) = 1/9$ y-int

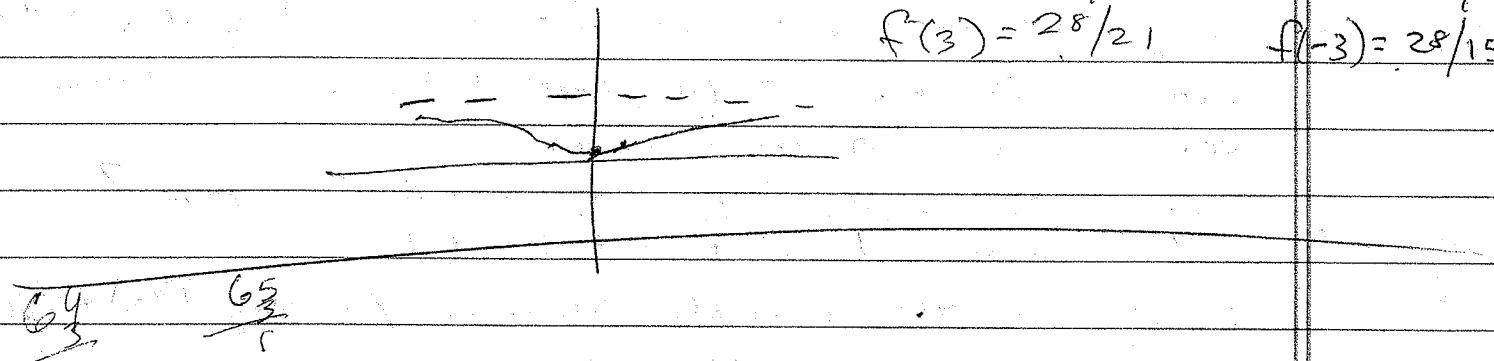
HA = 3/1 no roots no VA

$f \neq f(-x)$ $f(-x) = \frac{3x^2+1}{x^2-x+9}$ not odd or even

$f(1) = 4/11$ $f(-1) = 4/9$

$f(2) = 13/15$ $f(-2) = 13/11$

$f(3) = 28/21$ $f(-3) = 28/15$



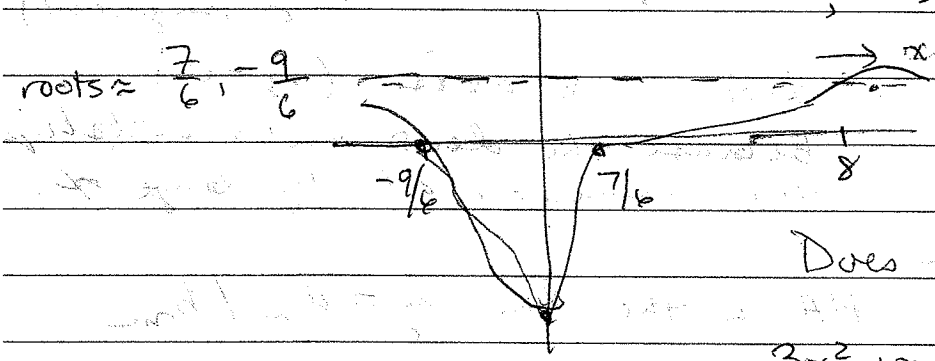
64/3 65/3

12. $f(x) = \frac{3x^2+x-5}{x^2+1}$ $m = n \rightarrow HA = 3$

no VA, $3x^2+x-5 = 0 \rightarrow$ ~~$(3x-5)(x+1)$~~

$x = \frac{-1 \pm \sqrt{1-4(3)(-5)}}{2 \cdot 3} = \frac{-1 \pm \sqrt{61}}{6}$

roots $\approx \frac{7}{6}, -\frac{9}{6}$



$f(0) = -5$

$f(10) = 305/101$

Does $f(x) = 3$ anywhere?

$3x^2+x-5 = 3(x^2+1)$

$3x^2+x-5 = 3x^2+3$

@ $x = 8$ $y = 3$ (crosses)