Thursday - Use these ideas to solve log eqns:

Ideas 10.1.2 & 10.2.5 & "one-oneness" "1-1-ness"

If $x = y \overset{\longleftrightarrow}{\text{then}} \log x = \log y$

If $x = y \overset{\longleftrightarrow}{\text{then}} a^x = a^y$

Use these to "strip" the log off an expression to solve for x

As well as the properties 1-7 and the change of base laws for logs & exponentials:

$$\log_a x = \frac{\log_b x}{\log_b a} \quad \& \quad a^x = b^{x \log_b a}$$

$$a^x = b^{\log_b a^x}$$

Also, it's helpful to know the natural log versions of these:

$$\log_a x = \frac{\ln x}{\ln a} \quad \& \quad a^x = e^{x \ln a}$$

Also, in $a^x = b^{\log_b a^x}$, if $x = 1$ then it "reduces" to $\boxed{a = b^{\log_b a}}$ (used in Ex. 10.2.20)

Use this to strip an antilog from ~~an~~ a log expression.

This is what the book means by "using exps to undo logs."

Also called the "raising both sides" operation.

Ex of quadratic in disguise:

$\cancel{2\cdot 3^2}$

$\to 2(3^{2x}) - 3^x - 3 = 0$

famous u-substitution: let $u = 3^x$

b/c $3^{2x} = (3^x)^2 = u^2$

Rewrite as $2\cdot u^2 - u - 3 = 0$

$(2u-3)(u+1) = 0$

$u = 3/2, -1$

Go back to $u = 3^x$:

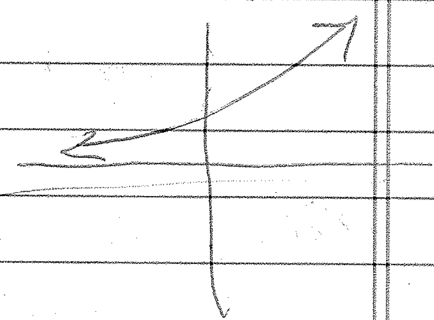
$\frac{3}{2} = 3^x$

$\log_3 \frac{3}{2} = \log_3 3^x$

$\log_3 \frac{3}{2} = x$

good b/c it's in the dom of 3^x

$-1 = 3^x$ no soln



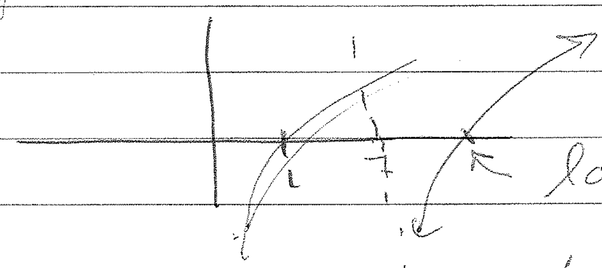
Sec 10.2

#7) $f(x) = \log_3(x-7) - \log_3(x+2)$

Before we solve, what's the dom f?

You need the most restrictive domain of the 2 terms.

dom: $\log(x-7) \Rightarrow x-7>0 \Rightarrow x>7$



$\log_3(x-7) = 0$

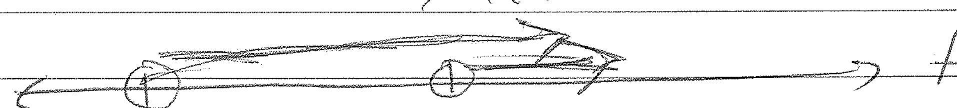
when $3^0 = x-7$

$1 = x-7$

$\underline{x = 8}$

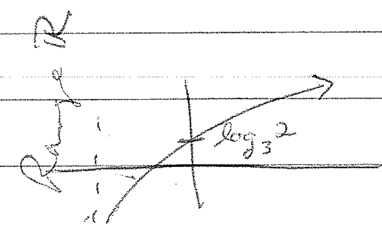
dom: $\log(x+2) \Rightarrow x+2>0$

$\Rightarrow x>-2$



$(+7, \infty)$

$\{x \in \mathbb{R} \mid x>7\}$



#8 $f(x) = \log_3(x+2)$

dom: $x+2>0 \rightarrow x>-2$

$(-2, \infty)$

inverse: $y = \log_3(x+2)$

① use def of log: $3^y = x+2$

② switch x + y: $3^x = y+2$

so $y = 3^x - 2 = f^{-1}$

#9 f) $3\ln x + 4\ln y - 4\ln z$

Condense using props.

(1) $\ln x^3 + \ln y^4 - \ln z^4$

(2) $\ln x^3 y^4 - \ln z^4$

(3) $\ln \dfrac{x^3 y^4}{z^4}$

#10 g) $\ln \sqrt[5]{\dfrac{x^2}{y^3}}$

(1) get rid of $\sqrt{}$

$\ln \left(\dfrac{x^2}{y^3}\right)^{1/5}$

(2) $\frac{1}{5}\left(\ln \dfrac{x^2}{y^3}\right)$

(3) $\frac{1}{5}\left(\ln x^2 - \ln y^3\right)$

(4) $\frac{1}{5}\ln x^2 - \frac{1}{5}\ln y^3$

plog (5) $\frac{2}{5}\ln x - \frac{3}{5}\ln y$

$\ln \equiv \log_e$

Sigma = Summation notation

$\sum_{i=1}^{n} i = ?$ "The sum of the first n integers, starting at 1"

$$\sum_{i=1}^{n} i = 1 + 2 + 3 + \dots + n$$

so let $n = 4$, then $\sum_{i=1}^{4} i = 1+2+3+4$

Call $i = 1$ is the initial number

but ~~[illegible]~~ i is the "index"

$$\sum_{i=0}^{7} i^2 = 0^2 + 1^2 + 2^2 + \dots + 7^2$$

not typical, but you can start at any $n \in \mathbb{Z}^+$

$$\sum_{i=2}^{7} (i+2) = \cancel{(2+2) + (2+3) + (2+4) + (2+5) + (2+6) + (2+7)}$$

$$= (2+2) + (3+2) + (4+2) + (5+2) + (6+2) + (7+2)$$

The "i" is indexing the expression you're summing

$$\sum_{i=1}^{n} 2i = 2 + 4 + 6 + \cdots + 2n$$
$$= 2\cdot 1 + 2\cdot 2 + 2\cdot 3 + \cdots + 2\cdot n$$

Gauss' formula $\displaystyle\sum_{i=1}^{n} i = 1 + 2 + 3 + \cdots + n = \frac{n}{2}(n+1)$

ex let $n = 6$

$$\sum_{i=1}^{6} i = 1 + 2 + 3 + 4 + 5 + 6 = 21$$
$$= \frac{6}{2}(6+1)$$
$$= 3 \cdot 7$$
$$= 21$$

formula for the sum of the first n integers

HW Read Sec 2.4
& see email for
important formulas
& problems