Ch. 5.3 Geometric Interpretation of Parabola

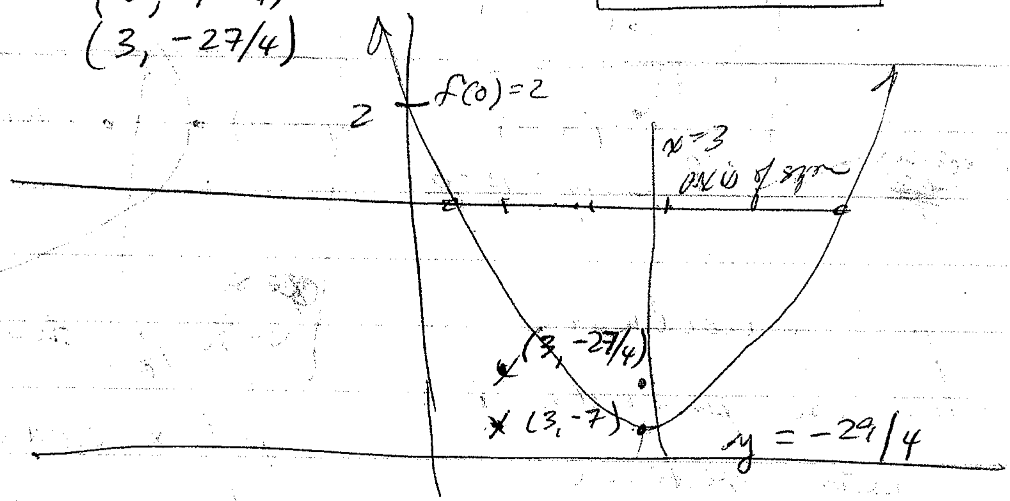
#4a) $f(x) = (x-3)^2 - 7 = x^2 - 6x + 2$

$y = \frac{1}{4p}(x-h) + k$: $\boxed{h=3,\ k=-7 \quad p = \frac{1}{4}}$

$4p = 1 \to p = 1/4$, $y = k - p$
$y = -1/4 + k$
$= -1/4 - 7$

focus: $(h, k+p)$
$(3, -7+\frac{1}{4})$
$(3, -27/4)$

directrix $\boxed{y = -29/4}$



Roots: $x = \frac{6 \pm \sqrt{36-8}}{2} = \frac{6 \pm \sqrt{28}}{2} = 3 \pm \sqrt{7}$

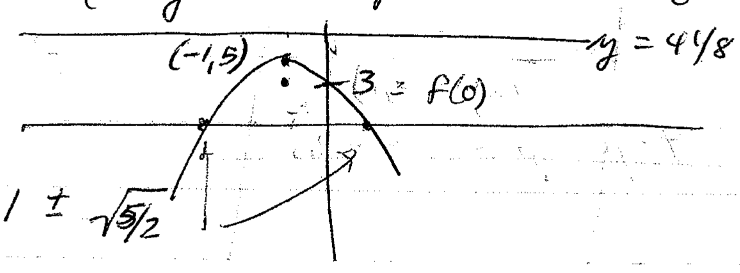
b) $f(x) = -2(x+1)^2 + 5$ $\qquad (h, k) = (-1, 5)$

$= \frac{1}{4p}(x-h)^2 + k$ $\qquad \frac{1}{4p} = -2 \to p = -\frac{1}{8}$

$y = -2(x^2 + 2x + 1) + 5$

focus $(h, k+p) = (-1, 5 + -\frac{1}{8}) = (-1, 39/8)$

directrix $y = k - p = 5 - -\frac{1}{8} = 41/8$



$x = -1 \pm \sqrt{5/2}$

#4c) $y = \frac{1}{16}(x-4)^2$

$= \frac{1}{4p}(x-h)^2 + k$

$(h, k) = (4, 0)$

focus $(h, k+p)$

$4p = 16$

$p = 4$

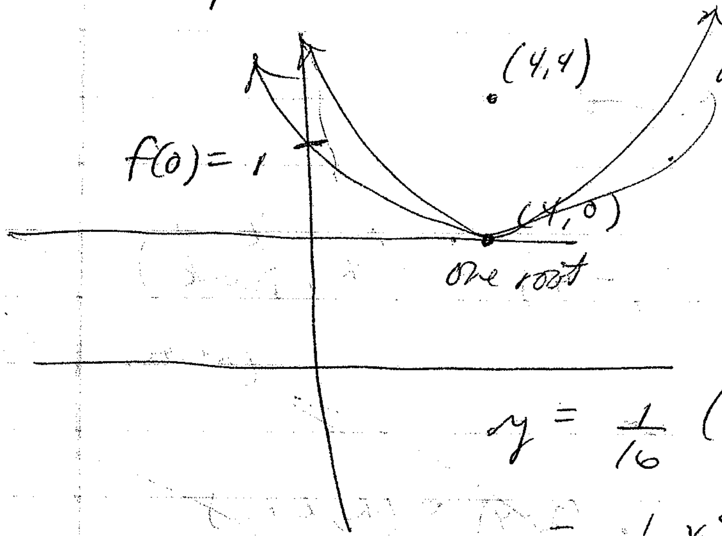
$(4, 0+4)$

$(4, 4)$

directrix $y = k - p$

$y = 0 - 4$

$y = -4$



$y = \frac{1}{16}(x^2 - 8x + 16)$

$= \frac{1}{16}x^2 - \frac{x}{2} + 1$

#5a) Focus $(3,2) = (h, k+p) = \cancel{(3, k+3)}$

Directrix $y = -4 = k - p$ $\quad \cancel{(3,2)}$

$\boxed{h = 3}$

$k + p = 2$

$k - p = -4$

$2k = -2$

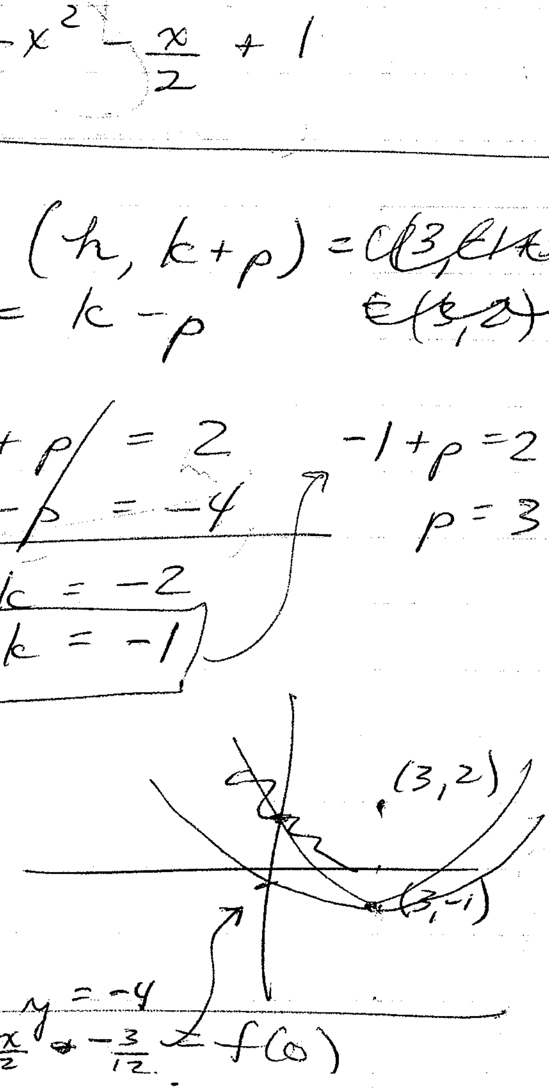
$\boxed{k = -1}$

$-1 + p = 2$

$p = 3$

axis of sym: $\boxed{x = 3}$

vertex $(3, -1)$



$y = \frac{1}{4p}(x-h)^2 + k$

$= \frac{1}{12}(x-3)^2 - 1 = \frac{x^2}{12} - \frac{x}{2} - \frac{3}{12} \leftarrow f(0)$

#5b) Focus: $(-1,3) = (h, k+p)$

Directrix $y = 7/2 = k - p$

$$\begin{aligned} 3 &= k + \cancel{p} \\ \tfrac{7}{2} &= k - \cancel{p} \\ \hline \tfrac{13}{2} &= 2k \end{aligned} \longrightarrow \boxed{k = \tfrac{13}{4}} \quad \boxed{h = -1}$$

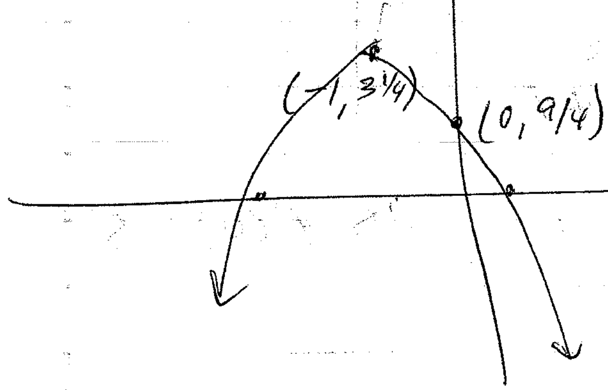
$$3 = \frac{13}{4} + p \qquad \boxed{p = -\frac{1}{4}}$$

$$y = \frac{1}{4p}(x-h)^2 + k$$

vertex $\left(-1, \frac{13}{4}\right)$

$$= -1(x+1)^2 + \frac{13}{4}$$

$$= -1(x^2 + 2x + 1) + \frac{13}{4} = -x^2 - 2x + 9/4$$



$$x = -1 \pm \sqrt{13/4}$$

$$= -1 \pm \frac{\sqrt{13}}{2}$$

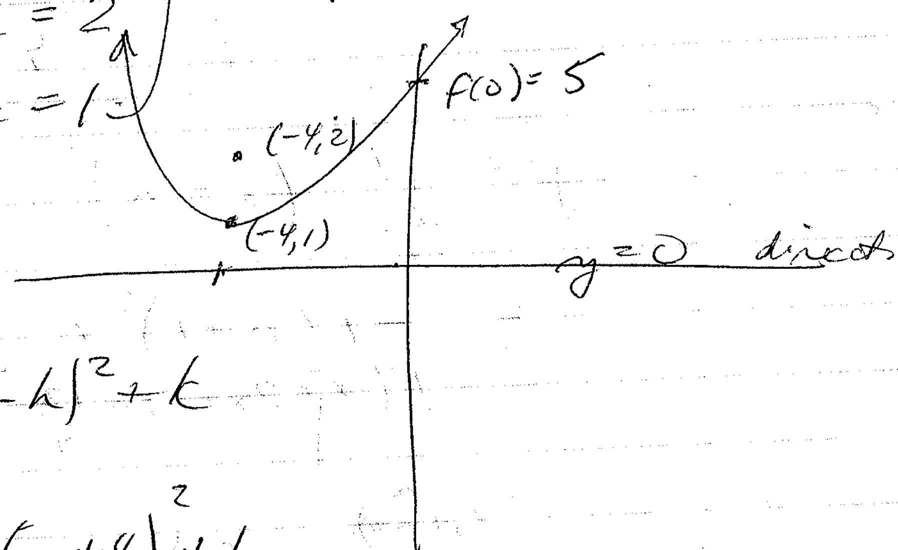
#5c) Focus $(-4, 2) = (h, k+p)$

Directrix $y = 0 = k - p$

$$\begin{aligned} k + \cancel{p} &= 2 \\ k - \cancel{p} &= 0 \\ \hline 2k &= 2 \\ k &= 1 \end{aligned}$$

$1 + p = 2, \quad \boxed{p = 1}$

$(h, k) = (-4, 1)$



$$y = \frac{1}{4p}(x-h)^2 + k$$

$$= \frac{1}{4 \cdot 1}(x+4)^2 + 1$$

$$y = \frac{1}{4}(x+4)^2 + 1 \longrightarrow f(0) = 5$$

#10 focus $\left(1, \frac{61}{12}\right) = (h, k+p)$

$(1, 5)$ vertex $= (h, k)$

$$k + p = \frac{61}{12}, \quad k = 5$$

$$p = \frac{61}{12} - 5 = \frac{1}{12}$$

$$y = \frac{1}{4 \cdot \frac{1}{12}}(x-1)^2 + 5 = 3(x-1)^2 + 5$$
$$= 3x^2 - 6x + 8$$

Directrix $\cancel{\&}\ y = k - p$ ∪

or

Focus $(h+p,\ k)$

Directrix $x = h - p$ ⊂

#9 $f(x) = \frac{1}{2}(x+3)^2 - 5$, y-int $= -\frac{1}{2}$

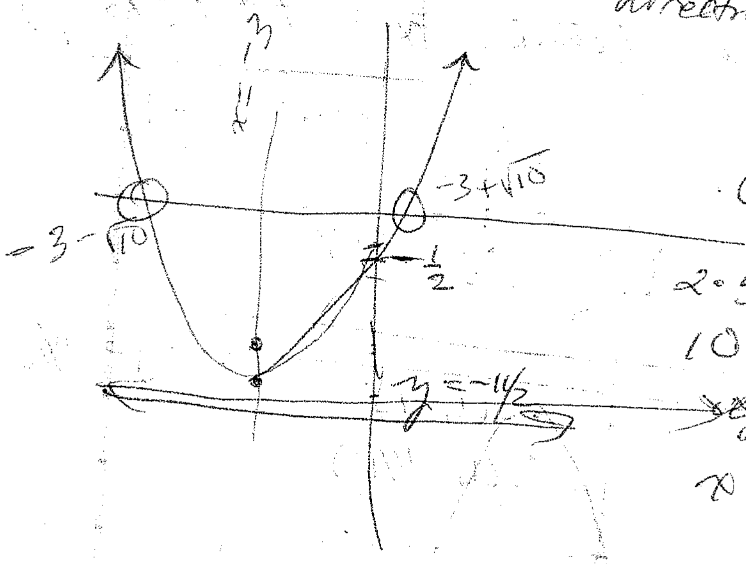
form: $y = \frac{1}{4p}(x-h)^2 + k$

vertex $(-3, -5)$

$p = \frac{1}{2}$

focus $(-3, -5+\frac{1}{2}) = (-3, -\frac{9}{2})$

directrix $y = -5 - \frac{1}{2} = -\frac{11}{2}$



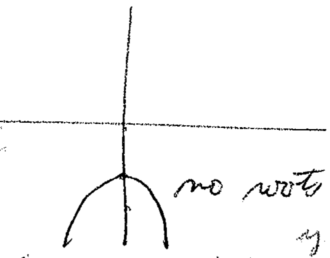
$0 = \frac{1}{2}(x+3)^2 - 5$

$2 \cdot 5 = (x+3)^2$

$10 = (x+3)^2 \rightarrow$ ~~reject~~

$x = -3 \pm \sqrt{10}$ ~~correct~~

#11 a) $x^2 + 18 = -6y$ given



$y = -\frac{1}{6}x^2 - 3$ isolate y

$= -\frac{1}{6}(x-0)^2 - 3 \longleftarrow$ template $y = \frac{1}{4p}(x-h)^2 + k$

read $h = 0,\ k = -3$ $\boxed{(0,-3) \text{ vertex}}$

solve for p $\frac{1}{4p} = -\frac{1}{6} \longrightarrow 4p = -6 \longrightarrow \boxed{p = -3/2 \text{ opens down}}$

focus $(h, k+p) = (0, -3 - 3/2)$

$\boxed{= (0, -9/2)}$

directrix $y = k - p = -3 + 3/2 = -3/2$

$\boxed{y = -3/2}$

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11 b) $-x^2 + 2x + 8 = y$ $\boxed{y\text{-int } (0,8)}$

$-(x^2 - 2x \quad\quad) + 8 = y$ complete the square

$-(x^2 - 2x + 1) + 8 = y - 1$

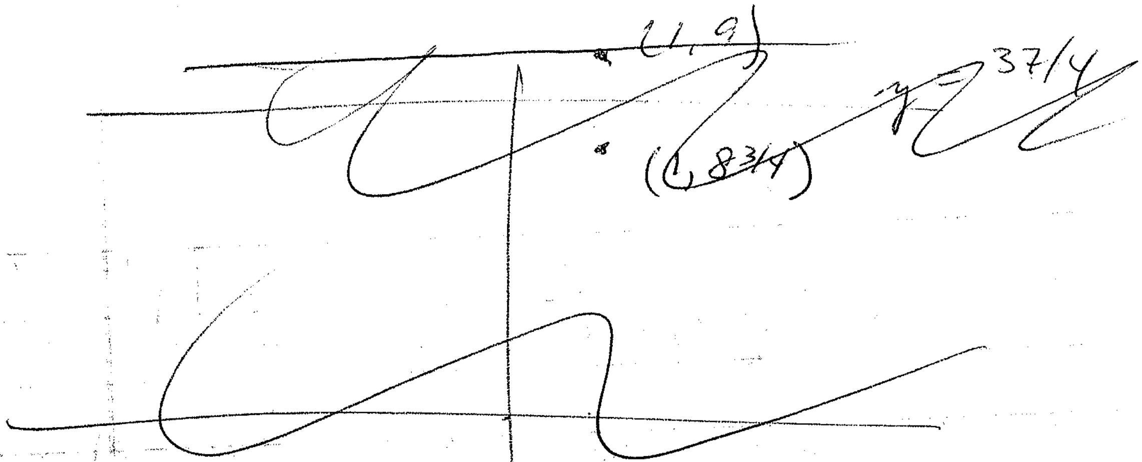
$-(x-1)^2 + 9 = y$ template form!

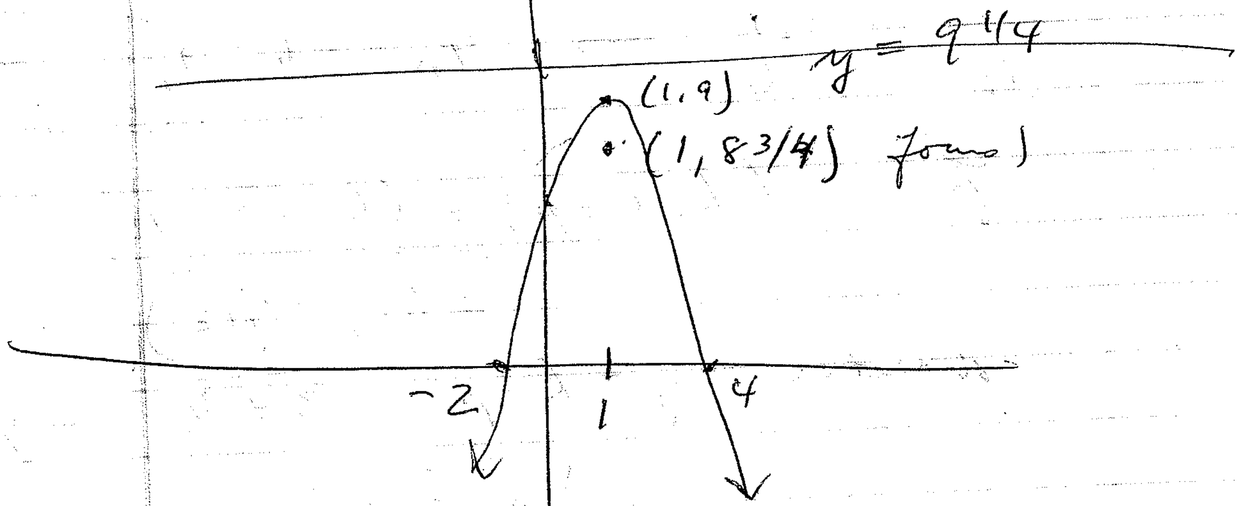
Seek this $y = -1(x-1)^2 + 9$ $h = 1, k = +9$

$\frac{1}{4p} = -1 \rightarrow \boxed{p = -\frac{1}{4}}$ $\boxed{(1,9) \text{ vertex}}$

focus $(h, k+p) = (1, 9 - \frac{1}{4}) = \boxed{(1, \frac{35}{4})}$

directrix $y = k - p = 9 + \frac{1}{4} = \boxed{37/4 = y}$





$y = -(x-1)^2 + 9$ roots 1 ± 3 : $(4, 0)$

$(-2, 0)$

4b $f(x) = -2(x+1)^2 + 5$ vertex $(-1, 5)$

$y = \frac{1}{4p}(x-h)^2 + k$ $h = -1$, $k = 5$

focus $(h, k+p) = (-1, 39/8)$ $\frac{1}{4p} = -2$

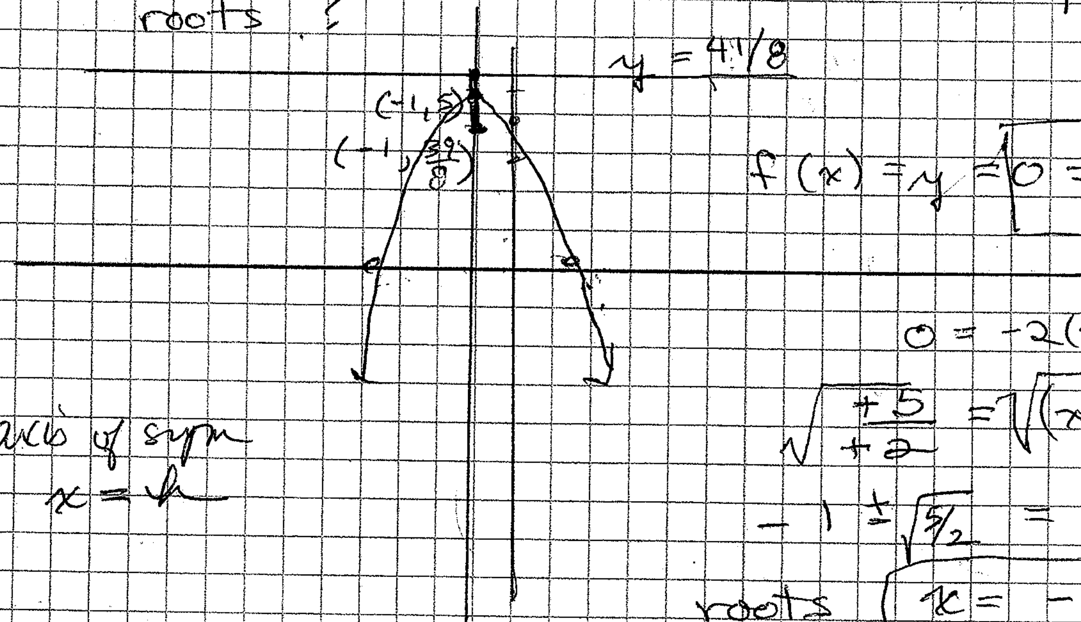
directrix $y = k - p = \frac{41}{8}$ $p = \frac{-1}{8}$

$k + p = +5 - \frac{1}{8} = \frac{39}{8}$

y-int: $f(0) = \cancel{5}\ 3$

$k - p = +5 - (-\frac{1}{8}) = +\frac{41}{8}$

roots ?



$f(x) = y = \boxed{0 = -2(x+1)^2 + 5}$

axis of sym $x = h$

$0 = -2(x+1)^2 + 5$

$\sqrt{\frac{+5}{+2}} = \sqrt{(x+1)^2}$

$-1 \pm \sqrt{5/2} = x + 1 - 1$

roots $\boxed{x = -1 \pm \sqrt{5/2}}$

p. 155

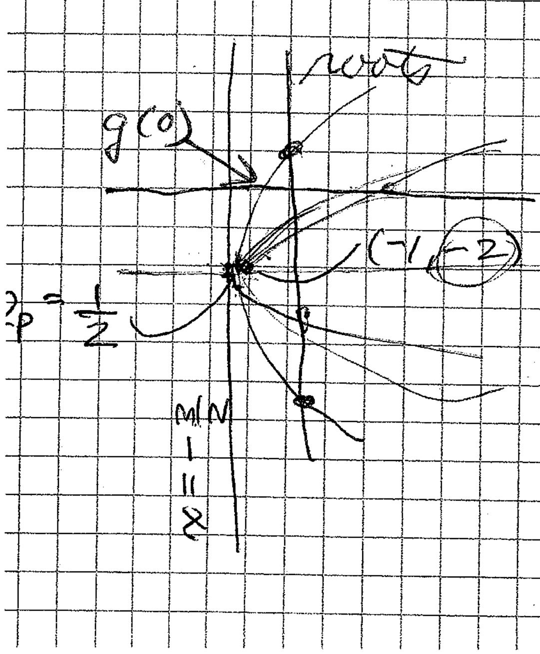
#1 focus $(-1, -2)$ dir: $x = -3/2 = -1\frac{1}{2}$

$(h+p, k)$

$\cancel{(-1, -2)}$

$2p = \frac{1}{2}$ geometrically

$y = -2$ $p = \frac{1}{4}$



$x = \frac{1}{4p}(y-k)^2 + h$

$x = 1(y+2)^2 + h$

$h + p = -1$ → $h = -\frac{5}{4}$, $k = -2$

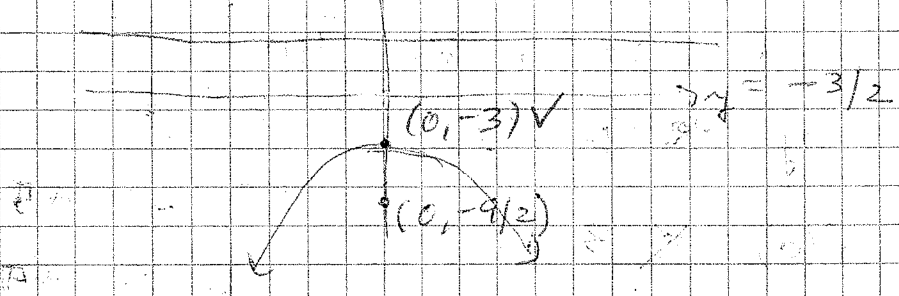
$h + \frac{1}{4} = -1$ vertex $(-5/4, -2)$

#11a)

Roots will be imag. Check!

$y = -\frac{1}{6}(x^2+18) = 0$

when $x^2+18 = 0 \rightarrow x = \pm 3i\sqrt{2}$

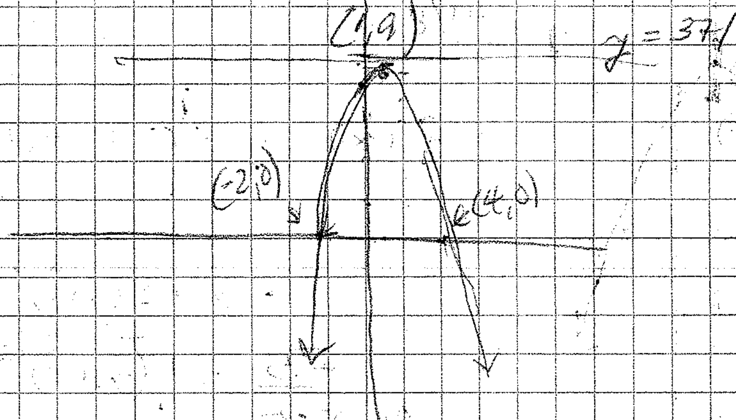


$\frac{1}{4p} = -\frac{1}{6}$

$6 = -4p$

$-\frac{6}{4} = p$

#11b)



Posed: lead coeff $= \frac{1}{16}$ → to get approx + position - find $f(0)$

wide bowl

$4p \rightarrow p = 4$

$\frac{1}{16} = \frac{1}{4p}$

$p = 4$