Skills review - key

Sheet #1

(1.) $\frac{3}{2a^3b^2} + \frac{5}{6a^2b}$ LCD $= 6a^3b^2$

Write $\frac{3}{3} \cdot \frac{3}{2a^3b^2} + \frac{ab}{ab}\,\frac{5}{6a^2b} = \boxed{\frac{9+5ab}{6a^3b^2}}$

(2.) $\frac{8}{9x^2-4} + \frac{5}{3x-2} - \frac{4}{3x+2}$ Factor $9x^2-4$ before finding LCD

$\frac{8}{(3x-2)(3x+2)} + \frac{5}{3x-2} - \frac{4}{3x+2}$ LCD $=(3x-2)(3x+2)$

$= \frac{8}{(3x-2)(3x+2)} + \left(\frac{3x+2}{3x+2}\right)\left(\frac{5}{3x-2}\right) - \left(\frac{3x-2}{3x-2}\right)\left(\frac{4}{3x+2}\right)$

$= \frac{8+15x+10-12x+8}{(3x-2)(3x+2)} = \boxed{\frac{3x+26}{(3x-2)(3x+2)}}$

(3) $\frac{3}{x+5} + \frac{2x}{x^2+3x-10}$ LCD seen from factoring $x^2+3x-10 = (x+5)(x-3)$ is the right

$= \left(\frac{x-3}{x+3}\right)\left(\frac{3}{x+5}\right) + \frac{2x}{(x+5)(x-3)}$

$= \frac{3x-9+2x}{(x-3)(x+5)} = \boxed{\frac{5x-9}{(x-3)(x+5)}}$

(4) $\frac{8}{r^2} - \frac{2r}{st} + \frac{t}{rs^2}$ LCD $= r^2s^2t$

$$\frac{s^2t}{s^2t}\cdot\frac{8}{r^2} - \frac{2r}{st}\cdot\frac{r^2s}{r^2s} + \frac{t}{rs^2}\cdot\frac{rt}{rt}$$

$$= \boxed{\frac{8s^2t - 2r^3s + rt^2}{r^2s^2t}}$$

No common factors to reduce by.

(5) $\frac{x^2+x-6}{x^2+7x+12} \div \frac{x^2-3x+2}{x^2+6x+8}$ invert + (x)

$$= \frac{\cancel{(x-2)}\cancel{(x+3)}}{\cancel{(x+4)}\cancel{(x+3)}}\cdot\frac{\cancel{(x+4)}(x+2)}{\cancel{(x-2)}(x-1)} = \boxed{\frac{x+2}{x-1}}$$

(6) $\frac{4x^3+2x^2-10x+1}{x-2}$ by long division

$$4x^2+10x+20+\frac{41}{x-2} \leftarrow \text{ANS}$$

$$\begin{array}{r} x-2\,\overline{)\,4x^3+2x^2-10x+1} \\ -(4x^3-8x^2) \\ \hline 10x^2-10x \\ -(10x^2-20x) \\ \hline 20x+1 \\ -(20x-40) \\ \hline 41 \end{array}$$

7. $x=2$ is not a root because $x-2$ is not a factor of $P(x)$ (the long poly)

We know this b/c there's a remainder.

Also, $P(2) \neq 0$, so $x=2$ cannot be a root and $x-2$ cannot be a factor.

Skill Sheet #2

(1.) $$\frac{10}{5x+3} = \frac{2}{10x-3}$$

$$(10x-3)\,10 = (5x+3)\,2$$

$$20x - 30 = 10x + 6$$

$$10x = 36$$

$$x = 36/10 = 18/5$$

Dom $\{x \in \mathbb{R} \mid x \neq -3/5, 3/10\}$

since $5x+3 \neq 0$

$\rightarrow x \neq -3/5$

and $10x - 3 \neq 0$

$\rightarrow x \neq 3/10$

$$\boxed{x = 18/5}$$

(2.) $$\frac{30}{x^2+5x+4} + \frac{10}{x+4} = \frac{4}{x+1}$$

Dom: $\{x \in \mathbb{R} \mid x \neq -1, -4\}$

LCD $= (x+4)(x+1)$

Multiply each term by LCD: to clear the denominators:

since $x^2+5x+4 \neq 0$

that is, $(x+4)(x+1) \neq 0$

so $x \neq -4, -1$

Gives: $30 + 10(x+1) = 4(x+5)$

$$30 + 10x + 10 = 4x + 20$$

$$6x = -20$$

$$x = -20/6 = -10/5$$

$$\boxed{x = -10/5}$$

Dom: $\{x \mid x \neq -3/2, -5\}$

(3) $$\frac{6}{2x - 3} = \frac{5}{x+5} + \frac{5}{2x^2+7x-15}$$

Factoring last term's denom:

$(2x+3)(x-5)$ no

$(2x-3)(x+5)$ yes

$2x+3 \neq 0$

$x \neq -3/2$

$x+5 \neq 0$

$x \neq -5$

Multiply by LCD – all terms:

$$6(x+5) = 5(2x-3) + 5$$

$$6x + 30 = 10x - 15 + 5$$

$$40 = 4x$$

$$\boxed{x = 10}$$

(4) $\dfrac{6}{x-3} - \dfrac{1}{x+3} = \dfrac{51}{x^2-9}$ Dom: $\{x \mid x \neq \pm 3\}$

(x) by LCD $6(x+3) - 1(x-3) = 51$

$6x + 18 - x + 3 = 51$

$5x = 30 \longrightarrow \boxed{x = 6}$

(5) $\dfrac{4x}{2x-1} = 2 - \dfrac{1}{2x-1}$ Dom: $\{x \mid x \neq \frac{1}{2}\}$

$4x = 2(2x-1) - 1$

$4x = 4x - 2 - 1$

$0 \neq -3 \longrightarrow \boxed{\text{no solution}}$

(6) $f(x) = \dfrac{3-x^2}{x^3-2x^2+4}$, $f(-1) = \dfrac{3-(-1)^2}{(-1)^3-2(-1)^2+4} = \dfrac{2}{1}$

$= \boxed{2}$

(7) $P(x) = \dfrac{x-1}{2x-3}$, $P(-3/2) = ?$

$P(-3/2) = \dfrac{-3/2 - 1}{2(-3/2)-3} = \dfrac{-3/2 - 2/2}{-3-3} = \dfrac{-5/2}{-6}$

$= \boxed{\dfrac{5}{12}}$

(8) $g(x) = f(x) - h(x)$

$g(a) = f(a) - h(a)$

$f(1) = g(1) + h(1)$ since $f(x) = g(x) + h(x)$

$h(-2) = f(-2) - g(-2)$ since $h(x) = \cancel{g(x)} - f(x) - g(x)$

Skill Sheet #3

(1) $\dfrac{4+\sqrt{20}}{4} = \dfrac{4+\sqrt{4\cdot5}}{4} = \dfrac{4+2\sqrt{5}}{4} = \boxed{\dfrac{2+\sqrt{5}}{2}}$

(2) $\dfrac{9-\sqrt{300}}{6} = \dfrac{9-\sqrt{3\cdot100}}{6} = \boxed{\dfrac{9-10\sqrt{3}}{6}}$ done!

(3) $\dfrac{11 \pm \sqrt{-121}}{11} = \dfrac{11 \pm 11i}{11} = \boxed{1 \pm i}$

(4) $\dfrac{6 \pm \sqrt{-1000}}{10} = \dfrac{6 \pm \sqrt{-10\cdot100}}{10} = \dfrac{6 \pm 10i\sqrt{10}}{10}$

$= \boxed{\dfrac{3 \pm 5i\sqrt{10}}{5}}$

(5) $\dfrac{14}{\sqrt{2x}} \cdot \dfrac{\sqrt{2x}}{\sqrt{2x}} = \dfrac{14\sqrt{2x}}{2x} = \boxed{\dfrac{7\sqrt{2x}}{x}}$

(6) $\dfrac{9}{3-\sqrt{y}} \cdot \dfrac{3+\sqrt{y}}{3+\sqrt{y}} = \boxed{\dfrac{27+9\sqrt{y}}{9-y}}$ done

(7) $\dfrac{8a}{\sqrt[4]{a}} \cdot \dfrac{\sqrt[4]{a}\,\sqrt[4]{a}\,\sqrt[4]{a}}{\sqrt[4]{a}\,\sqrt[4]{a}\,\sqrt[4]{a}} = \dfrac{8a\sqrt[4]{a^3}}{a} = \boxed{8\sqrt[4]{a^3}}$

(8) (12) (from other side)

$\dfrac{5+9i}{1-i} \cdot \dfrac{1+i}{1+i} = \dfrac{5+5i+9i+9i^2}{1-i+i-i^2}$

FOIL

$= \dfrac{5+14i-9}{1-(-1)} = \dfrac{14i-4}{2} = \boxed{7i-2}$

(8) i^9 Use highest multiple of 4 such that $4n + m = 9$. We do this b/c

$$i^{4n} = (i^4)^n = (i^2 \cdot i^2)^n = (1 \cdot 1)^n = 1$$

That is, $i^{4n} = 1$.

So, $i^9 = i^{\overset{(4n+m)}{8+1}} = i^8 \cdot i^1 = 1 \cdot i = \boxed{i}$

(9) $i^{14} = i^{12+2} = i^{12} \cdot i^2 = 1 \cdot (-1) = \boxed{-1}$

(10) $\frac{5+2i}{3i} \cdot \frac{3i}{3i} = \frac{15i + 6i^2}{9i^2} = \frac{15i - 6}{-9}$

$= \frac{-3}{-3} \cdot \frac{-5i+2}{3} = \boxed{\frac{-5i+2}{3}}$

factoring out -3 top + bottom

(11) $\sqrt{-3}\sqrt{-11} = i\sqrt{3} \cdot i\sqrt{11} = i^2\sqrt{33} = \boxed{-\sqrt{33}}$

Notice you cannot combine by multiplication the negative radicands. This is because $\sqrt{-1}$ is imaginary, so by definition $\sqrt{-3} = \sqrt{-1}\sqrt{3} = i\sqrt{3}$

Likewise for $\sqrt{-11}$.

ANS
$7i-2$

(12) $\frac{5+9i}{1-i} \cdot \frac{1+i}{1+i} = \frac{5+5i+9i+9i^2}{1-i^2} = \frac{14i - 4}{2} = \boxed{7i-2}$

$9i^2 \to -9$, $i^2 \to -1$

Skill Sheet 4

(1.) $\sqrt{3x+9} - 12 = 0$

$\sqrt{3x+9} = 12$ square each side

$3x+9 = 144 \longrightarrow x = \frac{135}{3} = \boxed{45}$

Check: $\sqrt{3\cdot 45+9} = \sqrt{135+9} - 12 = \sqrt{144} - 12$

$= 12 - 12 = 0$

(2.) $\sqrt{2x-5} + 4 = 1$ Isolate $\sqrt{\ }$

$\sqrt{2x-5} = -3$ see how this has no soln? Continue anyway, to understand how it can appear to work out.

$2x-5 = 9$

$2x = 14$

$x = 7$?

Check $x=7$:

Into original: $\sqrt{2\cdot 7-5} + 4 = \sqrt{9} + 4 = 3+4$

But $3+4 \neq 1$

(3) $2\sqrt{2x+7} - 1 = x+4$ Isolate $\sqrt{\ }$

$2\sqrt{2x+7} = x+4+1$

$\sqrt{2x+7} = \frac{x+5}{2}$

square $2x+7 = \frac{x+5}{2} \cdot \frac{x+5}{2}$

$2x+7 = \frac{x^2+10x+25}{4}$

$4(2x+7) = x^2 + 10x + 25$

$8x + 28 = x^2 + 10x + 25$

$x^2 + 2x - 3 = 0$

$(x-1)(x+3) = 0 \qquad \boxed{x = 1, -3}$

Must check

Check $x=1$: $2\sqrt{2x+7} - 1$

$= 2\sqrt{2(1)+7} - 1 = 2\sqrt{2+7} - 1 = 5$

$x + 4 = 1 + 4 = 5$ checks

Check $x=-3$: $2\sqrt{2(-3)+7} - 1 = 2\sqrt{-6+7} - 1 = 1$

$-3 + 4 = 1$ checks!

$\boxed{\text{Both } x = 1, -3 \text{ are solns}}$

(4) $\sqrt{2x-5} + \sqrt{x+1} = 3$ square both sides (FOIL on left)

$(\sqrt{2x-5} + \sqrt{x+1})(\sqrt{2x-5} + \sqrt{x+1}) = 9$

$2x - 5 + 2\sqrt{(x+1)(2x-5)} + x + 1 = 9$

$3x - 4 - 9 = -2\sqrt{(x+1)(2x-5)}$ isolate $\sqrt{\ }$

$3x - 13 = -2\sqrt{(x+1)(2x-5)}$ square both sides

$(3x-13)^2 = 4(x+1)(2x-5)$

$9x^2 - 78x + 169 = 8x^2 - 12x - 20$

$x^2 - 66x + 189 = 0$

$(x-3)(x-63) = 0 \longrightarrow \boxed{x = 3, 63}$ Must check

$x = 3$: $\sqrt{2(3)-5} - \sqrt{3+1} \stackrel{?}{=} 3$

$\sqrt{6-5} - \sqrt{4} = 1 - 3 \neq 3$

$x = 63$: $\sqrt{2(63)-5} - \sqrt{63+1} \stackrel{?}{=} 3$

$\sqrt{126-5} - \sqrt{64} = \sqrt{121} - \sqrt{64} =$

$= 11 - 8 = 3$

checks

$\boxed{x = 63}$

(5.) $\sqrt[3]{x-2} = 3$ Cube both sides

$x - 2 = 3^3$

~~x = 29~~

$x = 27 + 2 = 29$

check $\sqrt[3]{29-2}$

$= \sqrt[3]{27}$

$= 3$ ✓

$\boxed{x = 29}$

(6.) $\sqrt[4]{4x+1} = 2$ Raise to 4th power

$4x + 1 = 2^4 = 16 \longrightarrow 4x = 15$

$x = 15/4$

check $\sqrt[4]{4\left(\frac{15}{4}\right)+1} = \sqrt[4]{15+1} = \sqrt[4]{16} = 2$ ✓

$\boxed{x = \frac{15}{4}}$

16 36
6 30

$3 \cdot 63$

$2x^2 - 5x + 2x - 5$

$8x^2 - 20x + 8x - 20$

$8x^2 - 12x - 20$

#7 $\sqrt{32}\,\sqrt{50} = \sqrt{16\cdot 2}\,\sqrt{25\cdot 2} = 4\cdot\sqrt{2}\cdot 5\sqrt{2}$

$= 20\cdot 2$

$= \boxed{40}$

8 $\sqrt{(x+7)^2} = \boxed{x+7}$

9 $\sqrt[3]{x^7 y^{12}} = \sqrt[3]{x^6 x y^{12}} = \sqrt[3]{x^6}\sqrt[3]{x}\sqrt[3]{y^{12}}$

$= \boxed{x^2 y^4 \sqrt[3]{x}}$

10 $\sqrt{54} + \sqrt{24} = \sqrt{27\cdot 2} + \sqrt{12\cdot 2}$

$= \sqrt{27}\sqrt{12}\underbrace{\sqrt{2}\sqrt{2}} = 2\sqrt{9\cdot 3}\,\sqrt{4\cdot 3}$

$= 2\cdot 3\cdot 2\sqrt{3}\sqrt{3}$

$= 12\cdot 3 = \boxed{36}$

11 $\dfrac{\sqrt{125x^6}}{\sqrt{5x^3}} = \sqrt{\dfrac{125}{5}x^3} = \sqrt{25x^3} = \boxed{5x\sqrt{x}}$

12 $\sqrt[3]{9x} = -6$ cube each side (notice a negative cubed is negative)

$9x = -216$

$x = -216/9 = -24$