Skill Sheet #6 Key

#1. a) $f(x) = 3x - 4$

$y = 3x - 4$

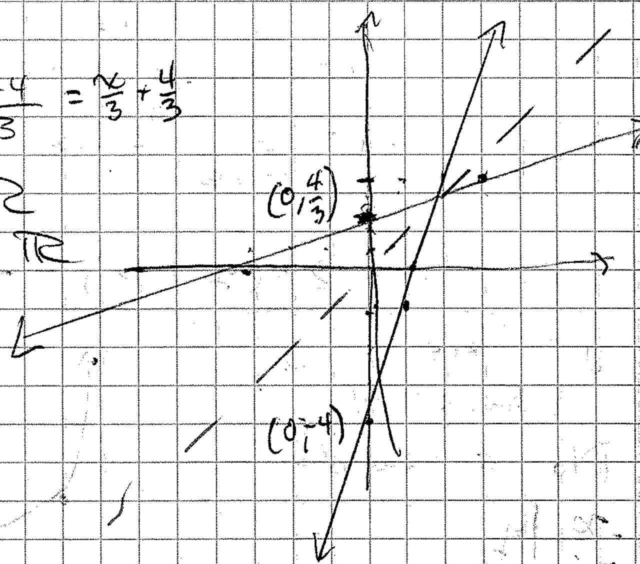
$x = 3y - 4$

$y = \frac{x+4}{3}$

$f^{-1}(x) = \frac{x+4}{3} = \frac{x}{3} + \frac{4}{3}$

Dom $f = \mathbb{R}$

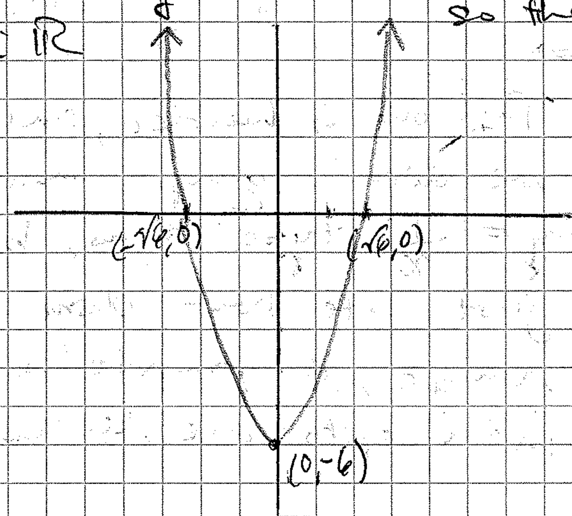
Dom $f^{-1} = \mathbb{R}$



c) $f(x) = x^2 - 6$ Dom: $x \in \mathbb{R}$

$y = x^2 - 6$ notice this is a parabola, hence not 1-1, so there's no inverse fcn. for it

Dom: $\mathbb{R}$



But, if we were to restrict the domain to the left or right, $(-\infty, 0]$ or $[0, \infty)$, then we could take the inverse.

$y = x^2 - 6$

$x = y^2 - 6$

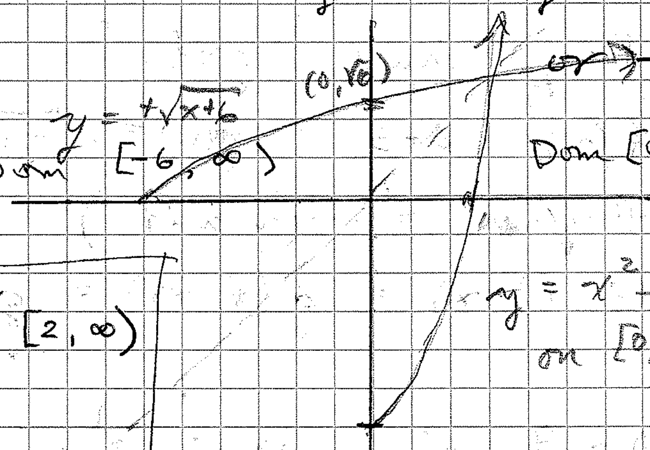
$x + 6 = y^2$

$y^2 = x + 6$

$\sqrt{y^2} = \pm\sqrt{x+6}$

$y = \pm\sqrt{x+6}$

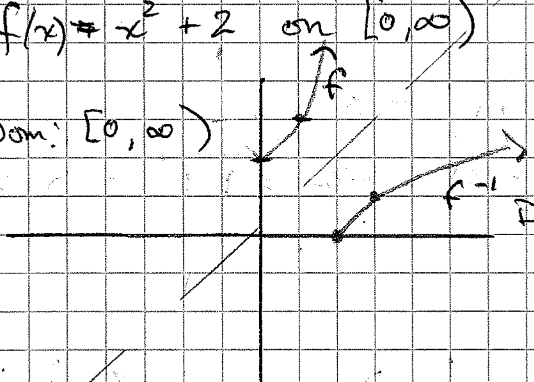
or $-\sqrt{x+6}$



Look at (e)

$f(x) = x^2 + 2$ on $[0, \infty)$

Dom: $[0, \infty)$



Dom: $[2, \infty)$

This has an inverse b/c it is 1-1. So, find $f^{-1}(x)$:

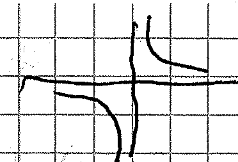
$y = x^2 + 2$

$x = y^2 + 2$

$y^2 = x - 2$

$y = \pm\sqrt{x-2}$

#1 d) $f(x) = \frac{2}{x+5}$ A hyperbola, like $f(x) = \frac{1}{x}$



which is 1-1, so invertible.

$y = \frac{2}{x+5} \longrightarrow x = \frac{2}{y+5} \longrightarrow xy + 5x = 2$

$\longrightarrow x(y+5) = 2 \longrightarrow y + 5 = \frac{2}{x} \longrightarrow y = \frac{2}{x} - 5$

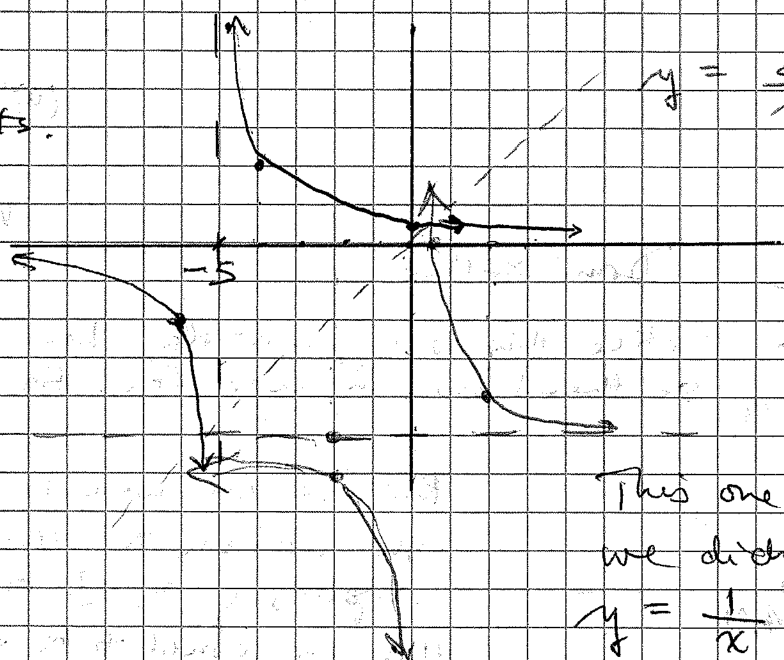
This is $f^{-1}(x)$

$y = 2/x+5$

Dom: $x \neq -5$

Plot a few pts.

x	y
-6	-2
-4	2
0	$2/5$
3	$1/4$



$y = \frac{2}{x} - 5$

Dom: $x \neq 0$

	x	y	
Range of f	-2	-6	Dom of f
	2	-4	
	$2/5$	0	
	$1/4$	3	

This one is harder, since we didn't graph many $y = \frac{1}{x}$ type fns, but just note how dom, range, asymptotes, intercepts are reversed.

#2a) $f \circ g$ and $g \circ f$

$f(x) = x - 3 \qquad g(x) = \sqrt{x+1}$

$(f \circ g)(x) = f(\sqrt{x+1}) = \sqrt{x+1} - 3$

$(g \circ f)(x) = g(f(x)) = g(x-3) = \sqrt{x-3+1} = \sqrt{x-2}$

b) $f(x) = 9 - x^2 \qquad g(x) = \frac{1}{3-x}$

$f \circ g = f(g(x)) = \cancel{f(9-x^2)}\ f\left(\frac{1}{3-x}\right) = 9 - \left(\frac{1}{3-x}\right)^2$

Simplify: $9 - \left(\frac{1}{3-x}\right)\left(\frac{1}{3-x}\right) = \boxed{9 - \frac{1}{9-6x+x^2}}$ ok

$g \circ f = g(f(x)) = g(9-x^2) = \frac{1}{3-(9-x^2)} = \boxed{\frac{1}{-6-x^2}}$ ok

#2c) $f(x) = \dfrac{7}{x+2}$ $\qquad g(x) = \dfrac{7}{x} + 2$ $\qquad$ Change this to $\dfrac{7}{x} - 2$

$$f \circ g = f(g(x)) = f\left(\frac{7}{x} + 2\right) = \frac{7}{\frac{7}{x} - 2 + 2} = \frac{7}{7/x} = \boxed{x}$$

$$g \circ f = g(f(x)) = g\left(\frac{7}{x+2}\right) = \frac{7}{7/(x+2)} - 2 = x + 2 - 2 = \boxed{x}$$

Notice f and g are thus ~~inverses~~, so
$f \circ g = x$ and $g \circ f = x$.

#3 a) $81^{3/4} = (81^{1/4})^3 = 2^3 = 8$

b) $27^{-3} = \dfrac{1}{27^3}$ Leave it ! I meant $27^{-1/3}$

$$27^{-1/3} = \frac{1}{27^{1/3}} = \frac{1}{3}$$

c) $(-1000)^{4/3} = \left[(-1000)^{1/3}\right]^4 = (-10)^4 = 10{,}000$

d) $(64)^{-5/6} = (64^{1/6})^{-5} = 2^{-5} = \dfrac{1}{2^5} = \dfrac{1}{32}$

e) $(-32)^{3/5} = \left[(-32)^{1/5}\right]^3 = (-2)^3 = -8$

f) $(-400)^{1/2} = ?$ This is an imaginary number

$$\sqrt{-400} = 20i$$

#4 Solve for x (where $b > 0$, $b \neq 1$)

a) $\log_b x = 0$, $\quad b^0 = x$, $\quad x = 1$
b) $\log_b b = x$, $\quad b^x = b$, $\quad x = 1$

$\Big\}$ Know these

c) $\log_{10} 10{,}000 = x$, $\quad 10^x = 10{,}000$, $\quad x = 4$

4d) $\log_7 x = 2$, $x = 7^2 = 49$

e) $\log_5 x = \frac{1}{5}$, $x = 5^{1/5}$ yikes!

How about $\log_5 \frac{1}{5} = x$, $5^x = \frac{1}{5}$, $x = -1$

f) $\log_e e^2 = x$, $e^x = e^2$, $x = 2$

g) $\ln x = -1$, $e^{-1} = x$, $x = 1/e$

h) $\ln e^7 = x$, $e^x = e^7$, $x = 7$

i) $\log_2 x = 6$, $x = 2^6$, $x = 64$

#5 a) $\log\left(\frac{x^3 y^2}{z}\right) = \log x^3 + \log y^2 - \log z$

$= 3\log x + 2\log y - \log z$

b) $\log_6 (x^3) = \log_6 x^3 = 3\log_6 x$

c) $\log_9 9^{x+2} = (x+2)\log_9 9 = (x+2)1 = x+2$

d) $\log 2 - \log 200 = \log \frac{2}{200} = \log \frac{1}{100} = -2$

b/c $10^{-2} = \frac{1}{100}$

e) $\log_7 10 + \log_7 (4.9) = \cancel{\log_7 10 + \log_7}$

$\log_7 (10)(4.9) = \log_7 49 = 2$ b/c $7^2 = 49$

f) $\frac{1}{3}(\log_8 x - \log_8 y) = \cancel{\frac{1}{3}\log_8 x - \frac{1}{3}\log_8 y}$

$= \frac{1}{3}\left(\log_8 \frac{x}{y}\right) = \log_8 \left(\frac{x}{y}\right)^{1/3}$

g) $\log\sqrt{2x} = \log(2x)^{1/2} = \frac{1}{2}\log 2 + \frac{1}{2}\log x$