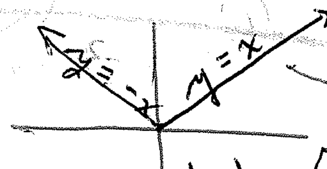
10/22



$$f(x) = |x| = \begin{cases} x, & x \geq 0 \\ -x, & x < 0 \end{cases}$$ BTW $|x| = \sqrt{x^2}$

→ $f(x) = |x| = c \longrightarrow x = c$ or $-c$

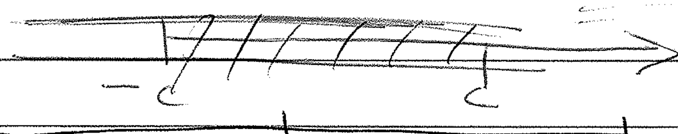
(book $-x = c$)

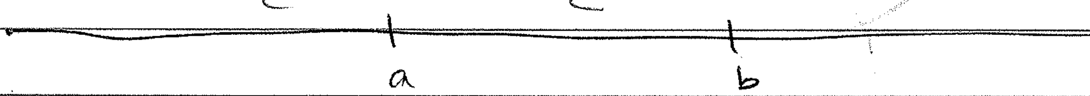
$|x| > c \longrightarrow x > c$ or $x < -c$

$(-\infty, -c) \cup (c, \infty)$

$|x| < c \longrightarrow x < c$ ~~or~~ and $x > -c$

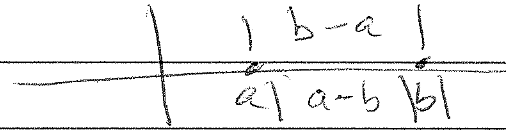
$(-c, c)$





☆ Def $|a-b|$ = distance between a + b

same $|a-b| = |b-a|$



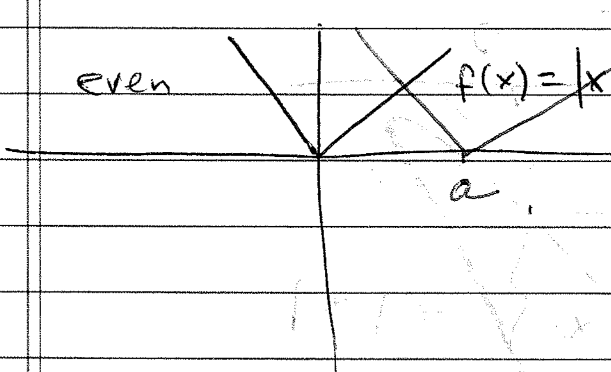
→ Def Triangle inequality – algebraic representation is not conducive to the geometric picture without vector math.

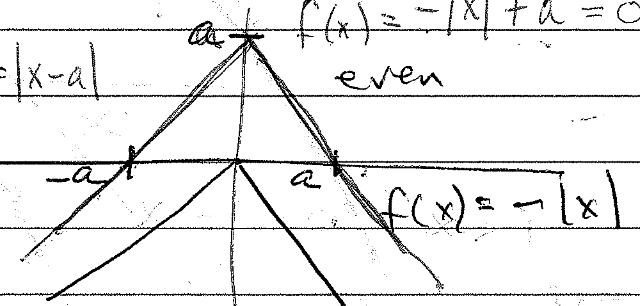
$$|a+b| \leq |a| + |b|$$

$$|a| - |b| \leq |a-b|$$

$$\boxed{\text{Also } |a| - |b| \leq |a+b| \leq |a| + |b|}$$

even

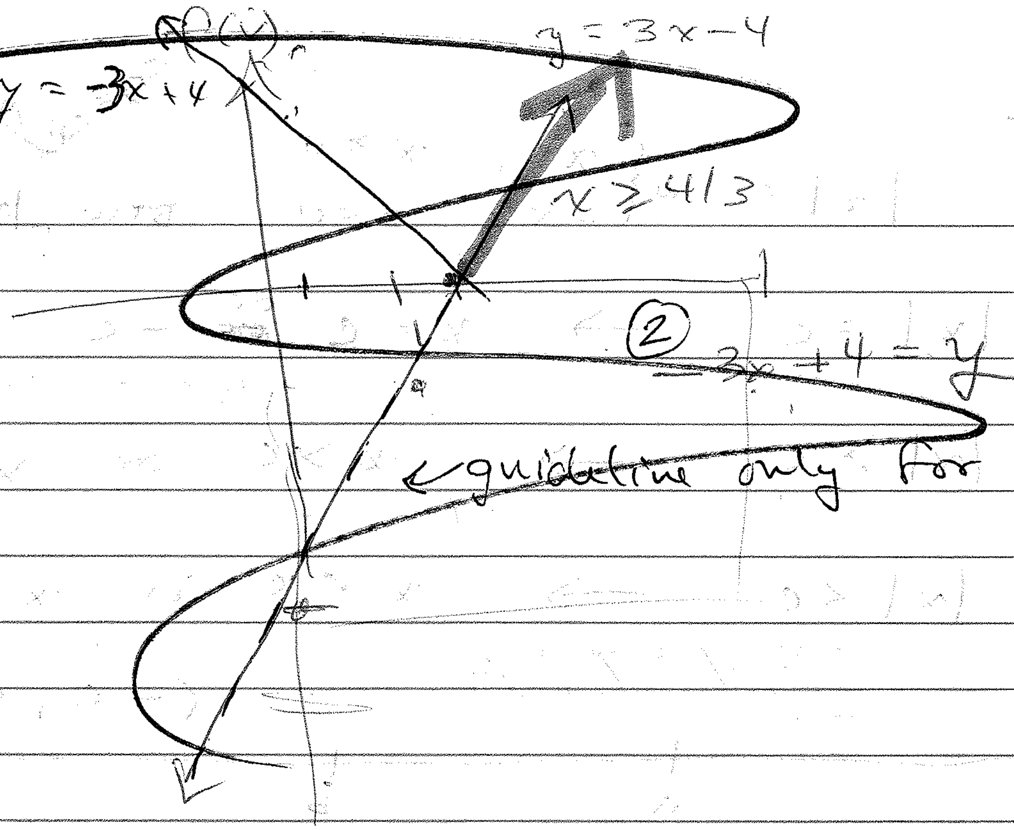




$f(x) \geq 0$ on $(-a, a)$

$f(x) < 0$ on $(-\infty, -a) \cup (a, \infty)$

Ex



$1 = |x| - x$

$1 < x$, $0 < 1 - x$, $(1-x)-$

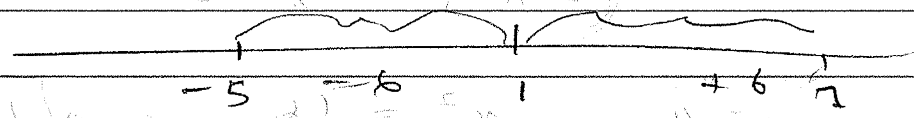
$1 \geq x$, $0 \geq 1 - x$, $1 - x = |x|$

$1 - x = |x|$ ← first isolate $|x|$

$1 = |x| - x$

$|x-1| < 6 \Rightarrow$ $x-1 < +6 \rightarrow x < 7$

$x-1 > -6 \rightarrow x > -5$



Triangle Inequality

$|a| + |b| \geq |a+b|$

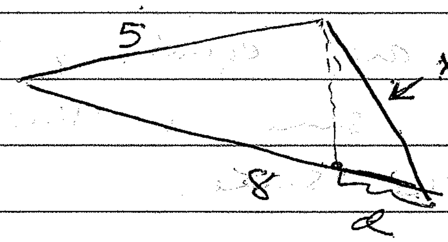
$a = 5 \quad b = -4$?

$|5| + |-4| \geq |5 + -4|$

$5 + 4 = 9 \geq 1$

Geometric

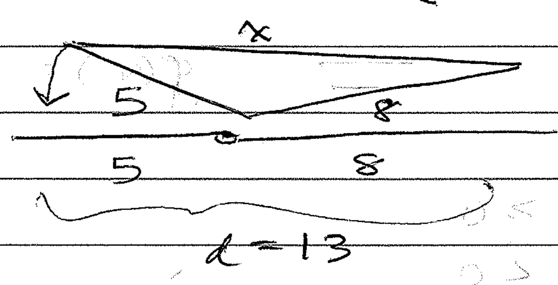
see link



$|8| - |5|$ $\quad d < x$

5

8



$d > x$

$|8| + |5| > x$

$|a| - |b| \leq |a+b| \leq |a| + |b|$

$$\sqrt{x^2} = \begin{cases} x, & x \geq 0 \\ -x, & x < 0 \end{cases}$$

$x = 4 \quad x^2 = 16, \quad \sqrt{16} = 4$

$x = -4 \quad (-x)^2 = (-4)^2 = 16$

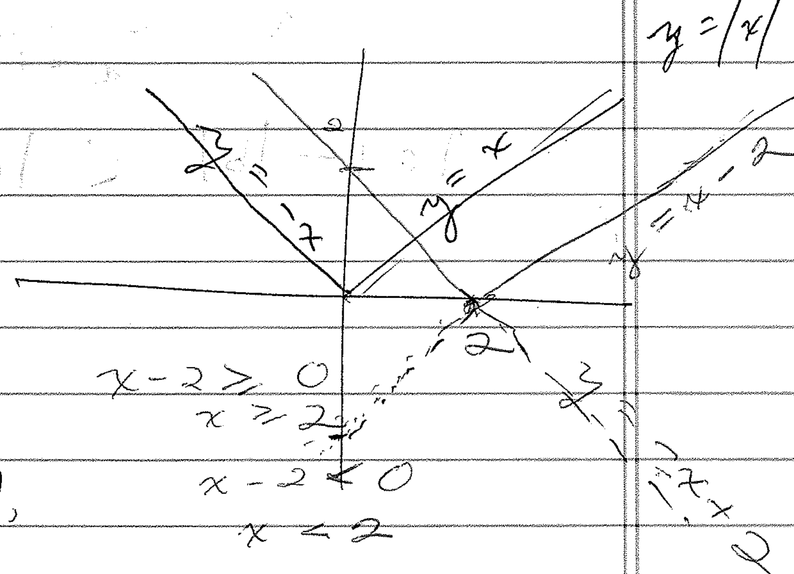
$\sqrt{(-4)^2} = 4 = -(-4)$

$$|x| = \begin{cases} -x, & x < 0 \\ x, & x \geq 0 \end{cases}$$

2 fcns are equal if their rules are the same & their domains are the same.

$f(x) = \sqrt{x^2} \quad = \quad g(x) = |x|$

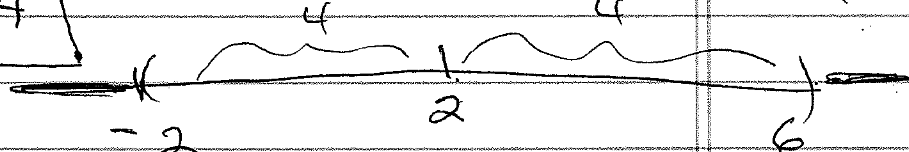
$$\begin{cases} x, & x \geq 0 \\ -x, & x < 0 \end{cases}$$



$y = |x - 2|$

$$y = \begin{cases} x - 2, & x - 2 \geq 0 \quad x \geq 2 \\ -(x-2), & x - 2 < 0 \quad x < 2 \end{cases}$$

$\boxed{|x - 2| > 4}$



Use the definition of $|x|$ to solve eqns in abs. value.

Numeric — Simplest: $|x| = 3 \rightarrow x = 3$ or $x = -3$

i.e. $\underbrace{-x = 3}$ if $x < 0$ or $\underbrace{x = 3}$ if $x \geq 0$

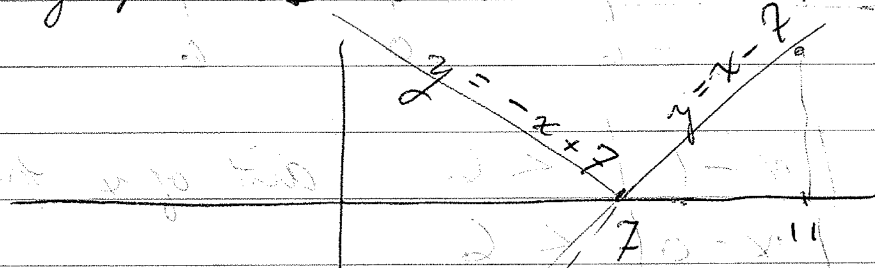
$x = \pm 3$

Functional

$$|f(x)| = \begin{cases} f(x) & \text{if } f \geq 0 \\ -f(x) & \text{if } f < 0 \end{cases}$$

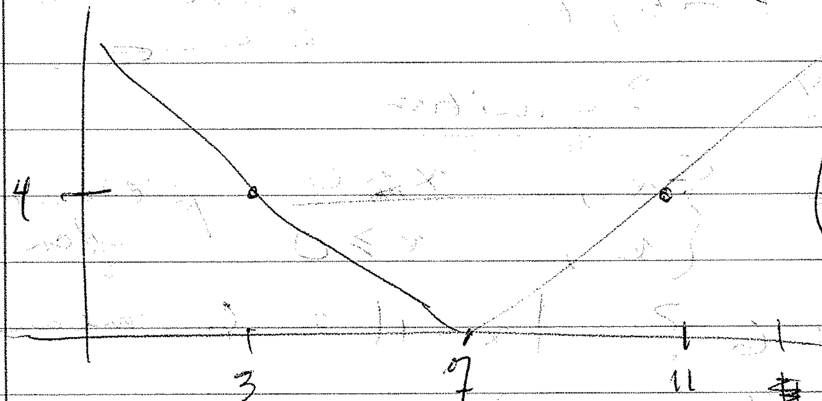
~~Thus,~~ if $f(x) = x - 7$ then $|f(x)| = |x-7|$

which graphs as seen:



Note that $|x-7| = 4$ means $x - 7 = 4$
(or $x = 11$) when $x - 7 \geq 0$
(or $x \geq 7$)

and $x - 7 = -4$ (or $x = 3$)
when $x - 7 < 0$ (or $x < 7$)

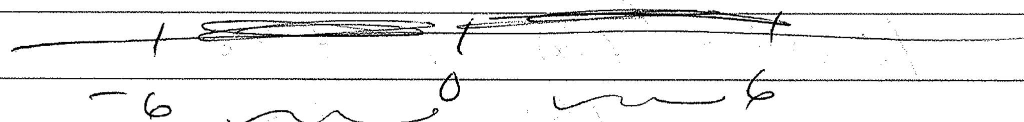


Alternatively:
$|x-7| = 4$ means
"the distance between
x & 7 is 4"
which is true for
$x = 11$ & $x = 3$.

Geometric interpretation of abs. value

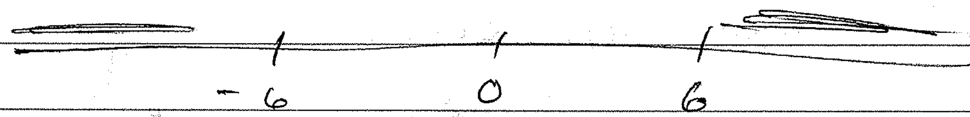
$|-6|$ dist of -6 from zero

$|6|$ " " 6 " "



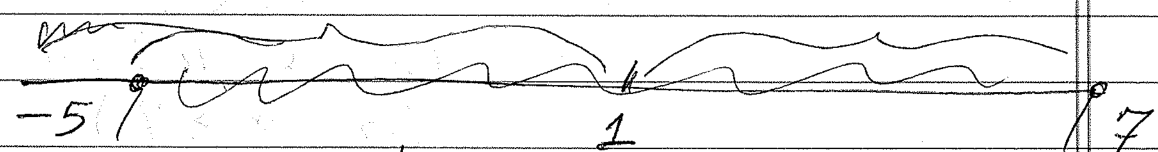
$|-6| = |6|$

$|x| > 6$ dist of x from zero is outside of 6



$|x-1| < 6$ dist of x from 1 is less than 6

$|x-0| < 6$



$|x-1| \leq 6 \not\leq |x-1| < 6$

$(-5, 7)$ geometric

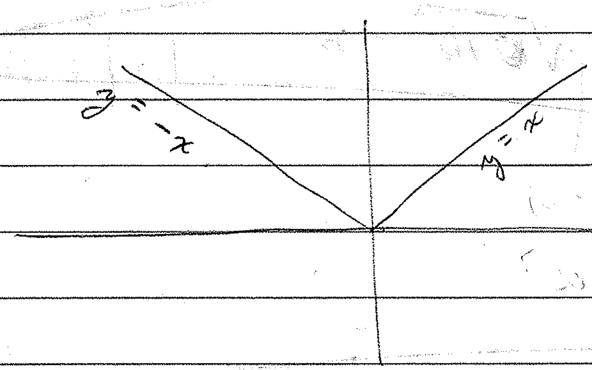
Algebraic? Definition

$$|x| = \begin{cases} -x, & x \leq 0 \\ x, & x \geq 0 \end{cases}$$ piecewise fcn.

$|x-1| < 6$? $|x-1| = 6$ means

either $|x-1| = \begin{cases} -(x-1) & \text{when } x-1 < 0 \\ x-1 & \text{when } x-1 \geq 0 \end{cases}$
or

Sc 7.2 $\quad f(x) = |x| = \begin{cases} +x, & x \geq 0 \\ -x, & x < 0 \end{cases}$



e.g. $|3| = 3$

$|-2| = -(-2)$
$= 2$

$f(x) = \sqrt{x^2} = \begin{cases} +x, & x \geq 0 \\ -x, & x < 0 \end{cases}$ e.g. $\sqrt{2^2} = 2$

$\sqrt{(-2)^2} = --2$
$= 2$

Hence, $|x| = \sqrt{x^2}$ (same rule, same domain)

Properties

- $|a| = |-a|$ $\qquad |3| = 3 = |-3|$ ✓
- $|a|^2 = a^2$ $\qquad |-4|^2 \overset{?}{=} 4^2$ ✓
- $|ab| = |a||b|$ $\qquad |-1 \cdot 7| \overset{?}{=} |-1||7|$

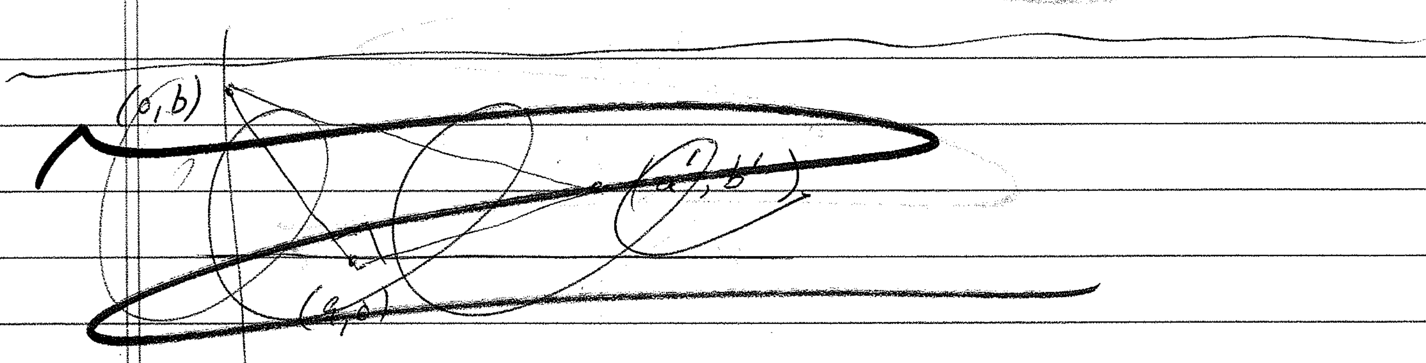
$\downarrow$

$|-7| \overset{?}{=} 1 \cdot 7$

$7 = 7$ ✓

- $\left|\frac{a}{b}\right| = \frac{|a|}{|b|}$ $\qquad \left|\frac{2}{-1}\right| \overset{?}{=} \frac{|2|}{|-1|}$

$|-2| = 2 \overset{?}{=} \frac{2}{1}$ ✓



(a,b)

$\sqrt{a^2+b^2} = d$

$|b| = \sqrt{0+b^2} = b$

$|a|^2 + |b|^2 = d^2$

$(0,0)$ $(a,0)$

$|a| = \sqrt{a^2+0} = a$

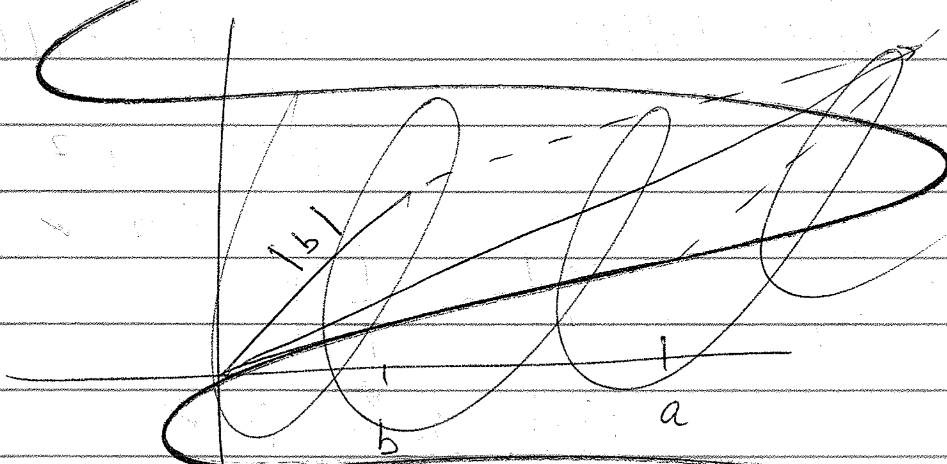
$(-a,0)$ — a — 0 — b — $(b,0)$

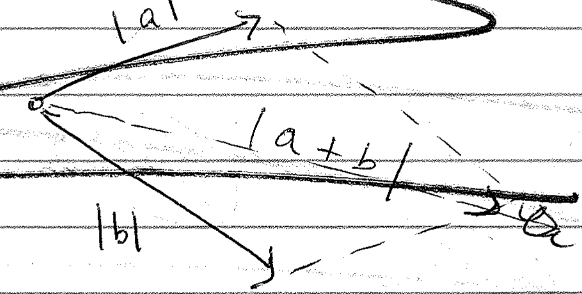
$|a| + |b| = b - -a = b + a$

$|a+b| = \sqrt{(a+b)^2} = \sqrt{a^2 + ab + b^2}$

$|a| + |b| = \sqrt{a^2} + \sqrt{b^2} =$

$\geq 0 \quad \geq 0$





Solve graphically strict inequality

Ex 7.1.8 $f(x) = \frac{-x}{x+8} > 3 \rightarrow \frac{-x}{x+8} - \frac{3}{1} > 0$ LCD

$\frac{-x-3x-24}{x+8} > 0$ $\boxed{x \neq -8}$

$\frac{-4x-24}{x+8} > 0$ Do NOT CROSS MULTIPLY

Part 1 $-4x-24 > 0$ and $x+8>0$

$\cap$

$x < -6$ $x > -8$

$(-8,-6)$

-8 -6 0

$\cup$

Part 2 $-4x-24 < 0$ and $x+8<0$

$\cap$

$-4x < 24$ $x < -8$

$x > -6$ $\emptyset$

$(-\infty,-8) \cup (-6,\infty)$

-8 -6

disjoint sets

Ans $\cancel{(-\infty,-8)}$ omit $\cup (-8,-6) \cup \cancel{(-6,\infty)}$ omit

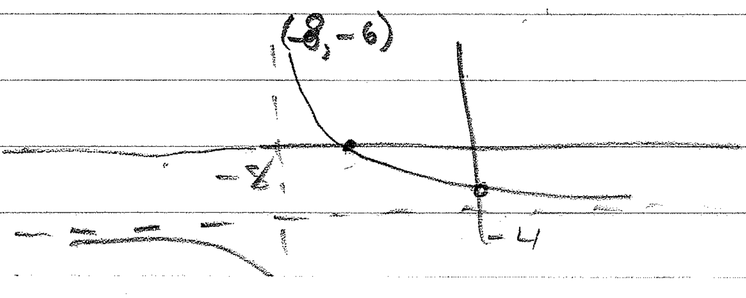
test pts

-8 -6

$f(x) = \frac{-4x-24}{x+8} = 0$ VA $x=-8$

root $x=-6$

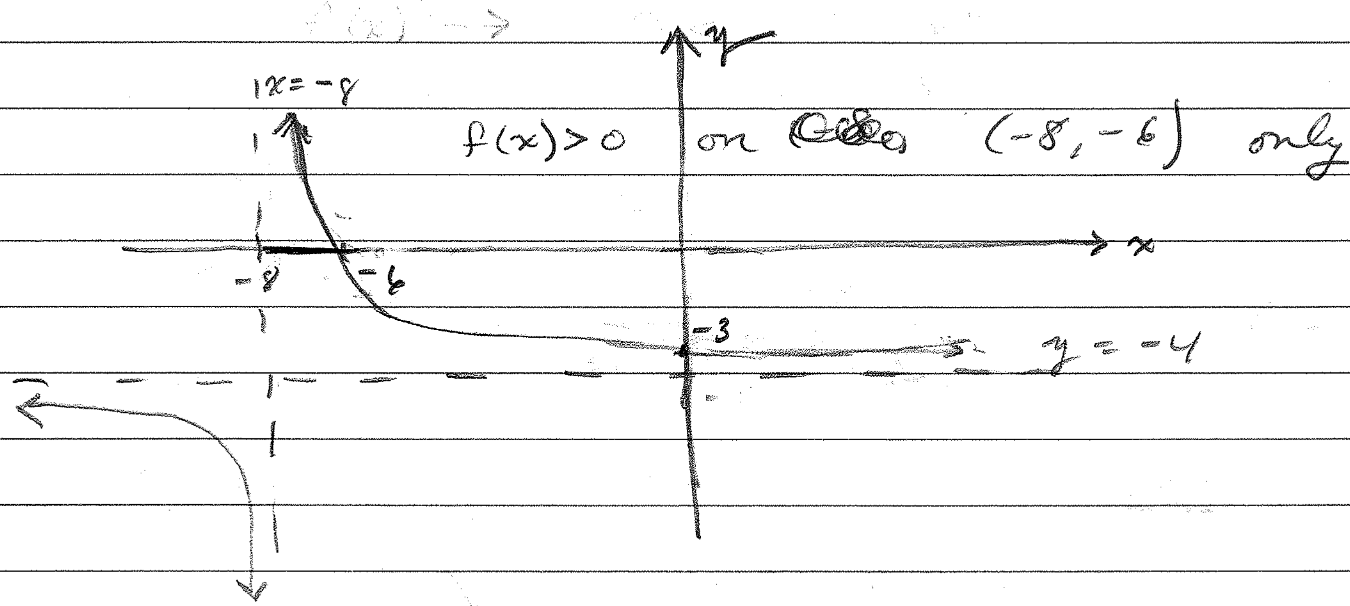
HA $y=-4$

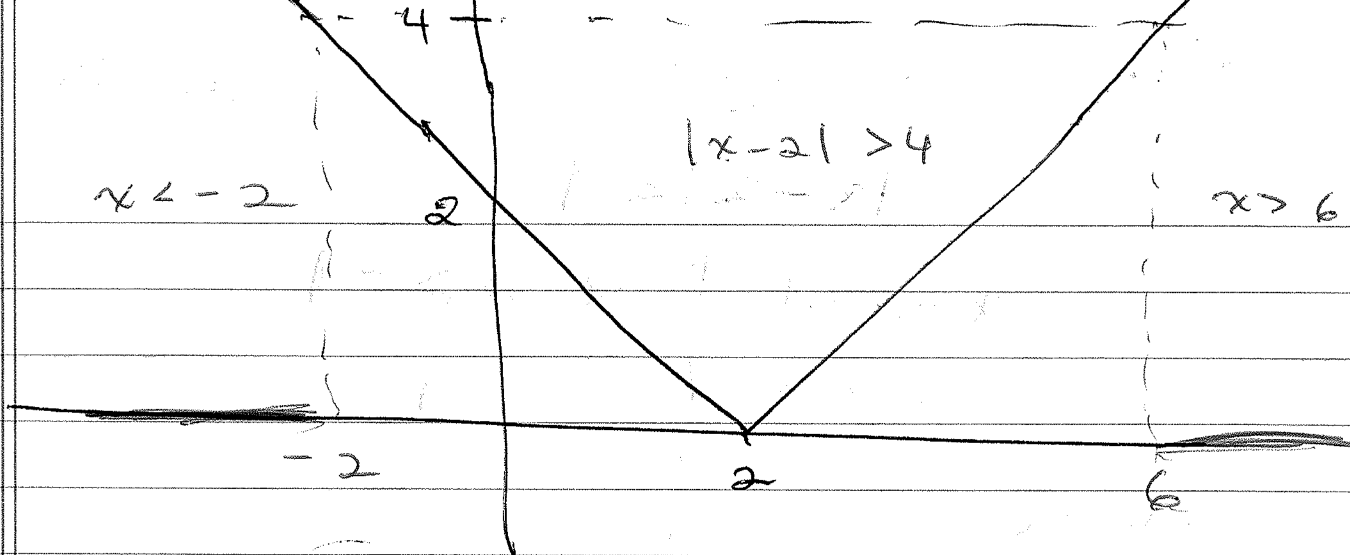


Graph $f(x) = \dfrac{-4x-24}{x+8} = -\dfrac{4(x+6)}{x+8}$

VA: $x = -8$ H.A: $y = -4$ $f(0) = -3$

root at $(-6, 0)$



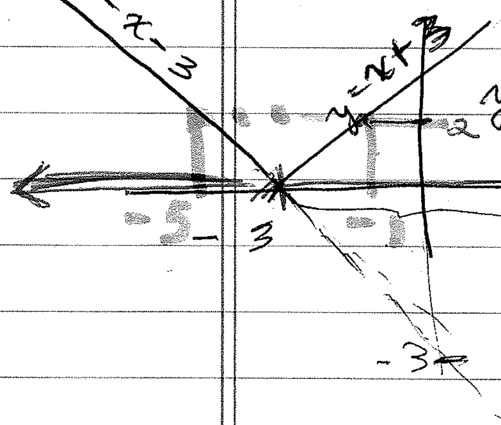


$f(x) = |x+3| < 2$ range

Graph $y = |x+3|$

Write the piecewise fcn $f(x) = |x+3|$

Name the intervals when this is true



$$y = |x+3| = \begin{cases} x+3, & x+3 \geq 0 \rightarrow x \geq -3 \\ -(x+3), & x+3 < 0 \rightarrow x < -3 \end{cases}$$

$$y = \sqrt{(x+3)^2}$$

Intervals

$|x+3| < 2$ $\qquad y = -x-3 \quad (-\infty, -3)$

$(-5, -1)$ $\qquad y = x+3 \quad (-3, \infty)$

$|x+3| < 2 \longrightarrow$ Distance formulation

$x+3 > -2 \qquad x+3 < 2$

$x > -5 \qquad x < -1$

$|x+3| < 2$

$|x - -3| < 2$

The dist x lies from -3 is 2 units.

$y = |x-2| \underset{-1}{+1} < \underset{-1}{2}$ Isolate $|x-2|$

$= |x-2| < 1$

$x-2<1$ | $x-2>-1$

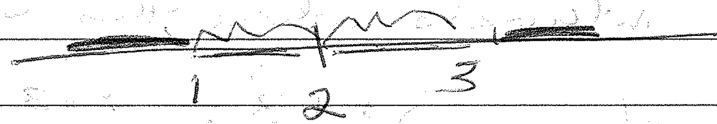
$x<3$ | $x>1$

~~Asvd~~

Solve $|x-2| = 1$

$x-2=+1$ | $x-2=-1$

$x=3$ | $x=1$ endpts of interval of inequality's solution



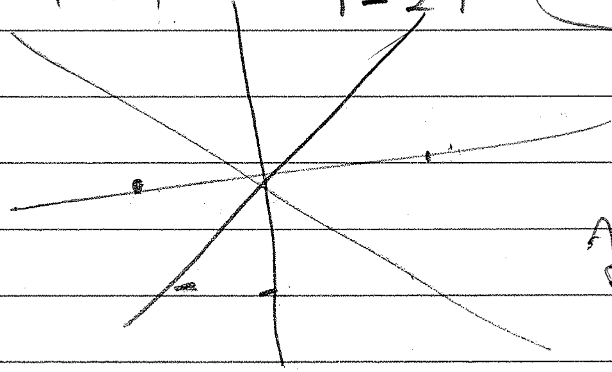
$|x-2|>1$

$x-2=-1$

$|y| = |x|$ $y=-2$

$|-2| = \left|\substack{2\\-2}\right|$

$y = \pm|x|$



$y = -|x|$