Ch. 2.4 Properties of sums + Special sums.

- $\sum_i ca_i = c \sum_i a_i$, c is a Constant

- $\sum_i a_i \pm \sum_i b_i = \sum_i (a_i \pm b_i)$

- $\sum_{i=m}^{n} c = (n-m+1)c$ because

$\sum_{i=1}^{n} c = nc$ and

there are $(n-m+1)$ terms in any sum $\sum_{i=m}^{n}$

For ex: $\sum_{i=1}^{30} 2 = 30 \cdot 2 = 60$

For ex: $\sum_{i=5}^{100} 2 = (100-5+1) \cdot 2 = 96 \cdot 2 = 192$

- $\sum_{i=1}^{n} (a_i - a_{i+1}) = a_1 - a_{n+1}$

(Telescoping sum)

Like #8b in hw and Ex 2.4.15, p. 58

- Change of index

$\sum_{i=1}^{n} a_i = \sum_{i=0}^{n-1} a_{i+1}$

Summary of topics + sample / example problems for Test 3, Chapter by Chapter:

Ch. 2.4 Sigma (summation) notation

Ch. 7.1 Inequalities
Ch. 7.2 Absolute Value
Ch. 7.3 Absolute Value Inequalities

Ch. 10.1 Exponential fcns.
Ch. 10.2. Logarithmic fcns.

Ch. 11.1 Systems of equations
Ch. 11.2 Systems of inequalities

$$\underline{Ex} \quad \sum_{i=1}^{16}(i+3) = \sum_{i=0}^{15}(i+1+3)$$

and hw # 3 a–c

For $\underline{ex}$ # 3a) $\displaystyle\sum_{i=1}^{7}(3i^2+2) = \sum_{i=0}^{6} ?$

Add $\underline{1}$ to i in the expression $3i^2+2$:

$$\sum_{i=0}^{6}\left(3(i+1)^2+2\right)$$

- $$\sum_{i=m}^{n} a_i = \sum_{i=1}^{n} a_i - \sum_{i=1}^{m-1} a_i$$

$\underline{\text{For ex}}$ $\displaystyle\sum_{i=15}^{40} i^2 = \sum_{i=1}^{40} i^2 - \sum_{i=1}^{16} i^2$

Use special sum $\displaystyle\sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6}$

on the right, twice:

$$\sum_{i=15}^{40} i^2 = \frac{40(41)(81)}{6} - \frac{16(17)(33)}{6}$$

Ch. 07.1 e Inequalities

Special Sums

- $$\sum_{i=1}^{n} i = \frac{n(n+1)}{2}$$

Ex $$\sum_{i=1}^{205} i = \frac{(205)(206)}{2} = (205)(103)$$ etc

- $$\sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6}$$

Ex $$\sum_{i=1}^{4} i^2 = \frac{4(5)(9)}{6} = \frac{180}{6} = 30$$

(true! $1 + 4 + 9 + 16 = 30$)

- $$\sum_{i=1}^{n} i^3 = \left[\frac{n(n+1)}{2}\right]^2$$ Know how to apply this, basically

Ex $$\sum_{i=1}^{7} i^3 = \left[\frac{7(8)}{2}\right]^2 = 28^2$$

Ch. 7.1 Inequalities

Study examples 7.1.4, 7.1.5, 7.1.6, 7.1.7 and 7.1.8 (showing that bringing x terms to one side + zero on other is the way to go!)

Ch. 7.2 Absolute Value Fcn

- Def: $|x| = \begin{cases} +x, & x \geq 0 \\ -x, & x < 0 \end{cases}$

  e.g. $|6| = 6$ since $6 \geq 0$

  $|-13| = 13 = -(-13)$ since $-13 < 0$

- Def $|x| = \sqrt{x^2}$

  e.g. $|3| = 3 = \sqrt{3^2}$

  $|-3| = 3 = \sqrt{(-3)^2}$

- $|a|^2 = a^2$

(
- $|a+b| \leq |a| + |b|$
- $|a-b| \geq |a| - |b|$

no hw or examples! Maybe ignore?
)

- Problems of each type:

$|f| = g$, #5e $\qquad |f| = |g|$, #5g $\qquad |f| = |g| + c$ #5h

- Graph of $f(x) = |x|$ and transformations (p. 208)
- Graph of $f(x) = |x^2|$ and transformations (p. 209)

see hw # 8 p. 216

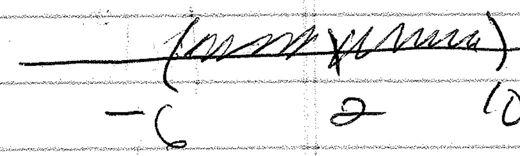
Ch. 7.3 This is like the quiz

$$|f(x)| < a \rightarrow -a < f(x) < a$$

ex $|x-2| < 8 \rightarrow -8 < x-2 < 8$

$$-6 < x < 10$$

Also, this means x is less than 8 units from 2

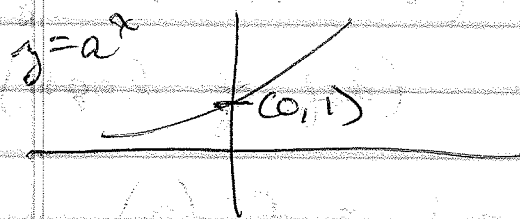


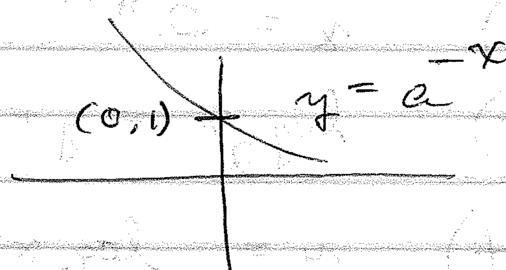
For Exs 7.3.1, p. 210, 7.3.2, p. 211

7.3.3 p. 212, and others to reinforce (see Ex 7.3.6 also)

Ch. 10.1 Exponential fcns

- Graphs of $y = a^x$ + transformations

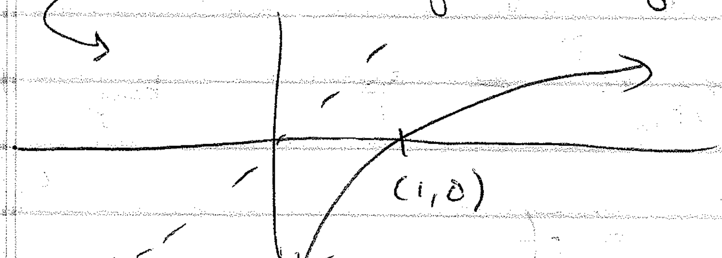




- Dom + range: $x \in \mathbb{R}$, $y > 0$ for all bases
- Ex pp 333-335 - sketches
- All 3 hw problems are good, esp. #2c the graph of $y = -3^x + 1$
- Also, Ex 10.15. p. 337 - solving an exp. eqn. for x using u-substitution

Ch. 10.2 Log fcns

- Def $\log_b a = x \iff a = b^x$
- Graph $y = \log_b x$ is the inverse fcn. of $y = b^x$



Dom + Range: $x > 0$
$y \in \mathbb{R}$
for all bases

$\underline{\text{Ex}}$ Finding f^{-1} of $f(x) = 2^{x+1}$

1) $y = 2^{x+1}$ (let $y = f(x)$)

2) $x = 2^{y+1}$ (switch $x + y$)

3) $\log_2 x = y+1$ (def of logs) *

4) $y = \log_2 x - 1 = f^{-1}(x)$

~~* You can also take log of both sides~~

* From 2) to 3) you could also say you took log of both sides

- Prop of logs + exs:

1. $\log_a a = 1$ by def $a^1 = a$

   $\underline{\text{Ex}}$ $\log_2 2 = 1$, $\ln e = 1$, $\log 10 = 1$

   $\ln$: natural log base e; $\log$: common log base 10

2. $\log_a 1 = 0$ by def $1 = a^0$, $a \neq 0$

3. $\log_a a^p = p$ by def. $a^p = a^p$!

   $\underline{\text{Ex}}$ $\log_4 4^7 = 7$

   $\underline{\text{Ex}}$ $\log_6 6^2 = 2$

(4.) $a^{\log_a m} = m$

This is mostly helpful in changing bases of an exp. fcn:

Ex p. 345 10.2.14

$f(x) = 2^x$ change to base e

$$2^x = e^{\ln 2^x} = e^{x \ln 2}$$

Ex HW #18 Change each to base e

a) $3^x = e^{\ln 3^x} = e^{x \ln 3}$

b) $6^x = e^{\ln 6^x} = e^{x \ln 6}$

c) $10^x = e^{\ln 10^x} = e^{x \ln 10}$

(5.) (6.) $\log_a mn = \log_a m + \log_a n$

$$\log_a (m/n) = \log_a m - \log_a n$$

(7) "plogam" $\log_a m^p = p \log_a m$

which has been used above a lot!

Any HW ~~or~~ example will do

⑧ Change of log base:

From base a: $\log_a x = \dfrac{\log_b x}{\log_b a}$ ← ~~[illegible]~~ to new base b

Again, any HW or example.
Esp. #15 p. 350

- Solving log eqns. – important ones

Ex 10.2.21 where we "strip" the log off as shown in class + in book

Ex 10.2.19 a "quadratic" in disguise

HW #22 d, f

- Check your solns. Watch the domain restrictions!

- ~~[illegible]~~ Evaluate w/o calculator #3, #19 p. 350