# Test 4 Key / Guide

1.a) $-130^\circ + 360^\circ = 230^\circ$

$-130^\circ - 360^\circ = -490^\circ$

b. ~~$\pi$~~ $\frac{11\pi}{6} + 2\pi = \frac{23\pi}{6}$

$\frac{11\pi}{6} - 2\pi = \frac{-\pi}{6}$

2. a) $90 - 57 = 33^\circ$ comp.

b) ~~$180 - 123 -$~~

$180 - 57 = 123$ supp

a) $\frac{\pi}{2} - \frac{2\pi}{5} = \frac{\pi}{10}$ comp

b) $\pi - \frac{2\pi}{5} = \frac{3\pi}{5}$ supp

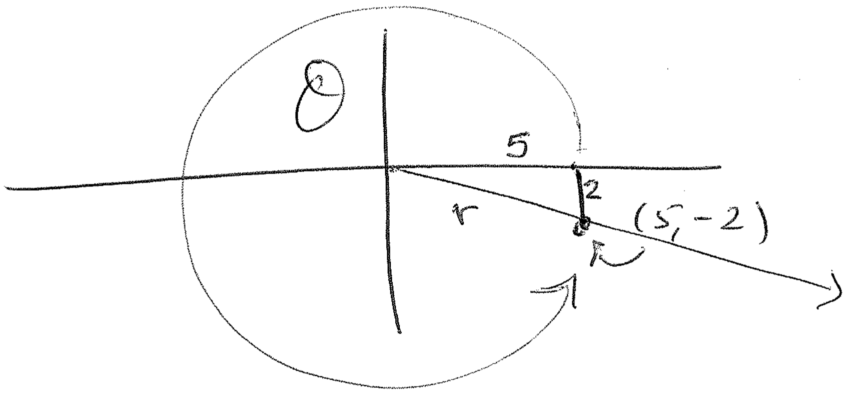
3. a) $115^\circ \to$ radian

$115^\circ \cdot \frac{\pi}{180^\circ} = \frac{23\pi}{36}$

b) 4 rad $\to$ deg

$4 \text{ rad} \cdot \frac{180^\circ}{\pi \text{ rad}} = \frac{720^\circ}{\pi}$ (Note, this is deg measure, where the # of deg is irrational, since it's $720/\pi$)

4. (a) Draw the $\theta$ in std position so you know the quadrant + reference triangle.




$r = \sqrt{5^2 + 2^2}$
$= \sqrt{29}$

a) $\sin\theta = \dfrac{-2}{\sqrt{29}}$ $\quad \cos\theta = \dfrac{5}{\sqrt{29}}$ $\quad \tan\theta = \dfrac{-2}{5}$

$\sec\theta = \dfrac{\sqrt{29}}{5}$ $\quad \csc\theta = -\dfrac{\sqrt{29}}{2}$ $\quad \cot\theta = \dfrac{-5}{2}$

b) Unit circle coordinates are as follows:

$\cancel{\left(\dfrac{\cos\theta}{r}, \dfrac{\sin\theta}{r}\right)}$ sorry

Since $r\cos\theta = 5$

then $1\cos\theta = \dfrac{5}{r} = \dfrac{5}{\sqrt{29}}$

unit circle form.

Also, $r\sin\theta = -2$

$1\sin\theta = \dfrac{-2}{r} = \dfrac{-2}{\sqrt{29}}$ $\qquad \left(\dfrac{5}{\sqrt{29}}, \dfrac{-2}{\sqrt{29}}\right)$

5. • Amplitude $= \dfrac{\max - \min}{2} = \dfrac{5-1}{2} = 2$ given from range [1, 5]

• Because its range is [1, 5], rather than [-2, 2], which would be the usual cosine, stretched w/ amp = 2, then must be a vertical shift 3 units. (from -2 to 1 and 2 to 5)

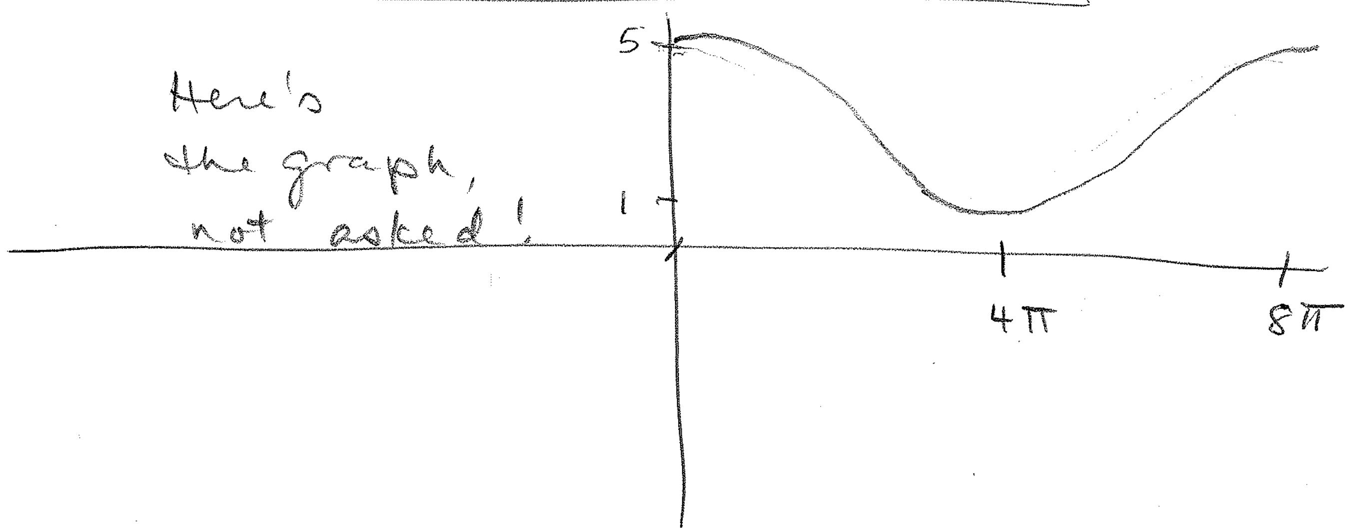
• So far: $y = 2\cos(\theta) + 3$

Now the $\theta$. Period $= 8\pi$; Form is $\cos Bx$,

where $B = \dfrac{2\pi}{\text{period}} = \dfrac{2\pi}{8\pi} = \dfrac{1}{4}$

So $\boxed{y = 2\cos\left(\frac{1}{4}x\right) + 3}$

Here's the graph, not asked!



10. $$\frac{\sin\theta}{1+\cos\theta} + \frac{1+\cos\theta}{\sin\theta} \stackrel{?}{=} 2\cos\theta$$

$$\text{LCD} = \sin\theta\,(1+\cos\theta)$$

$$\frac{(\sin\theta)(\sin\theta) + (1+\cos\theta)(1+\cos\theta)}{(1+\cos\theta)(\sin\theta)}$$

$$= \frac{\sin^2\theta + 1 + 2\cos\theta + \cos^2\theta}{(1+\cos\theta)(\sin\theta)} \qquad = 1$$

$$= \frac{1 + 1 + 2\cos\theta}{(1+\cos\theta)\sin\theta} = \frac{2 + 2\cos\theta}{(1+\cos\theta)\sin\theta}$$

$$= \frac{2\cancel{(1+\cos\theta)}}{\cancel{(1+\cos\theta)}\sin\theta} = \frac{2}{\sin\theta} = 2\csc\theta$$ ✓

11. $\dfrac{\cos(\alpha-\beta)}{\sin(\alpha)\sin(\beta)} \overset{?}{=} \cot(\alpha)\cot(\beta)+1$

Identity: $\cos(\alpha-\beta) = \cos\alpha\cos\beta + \sin(\alpha)\sin\beta$

So $\dfrac{\cos\alpha\cos\beta+\sin\alpha\sin\beta}{\sin\alpha\sin\beta} = \dfrac{\cos\alpha\cos\beta}{\sin\alpha\sin\beta} + \dfrac{\sin\alpha\sin\beta}{\sin\alpha\sin\beta}$

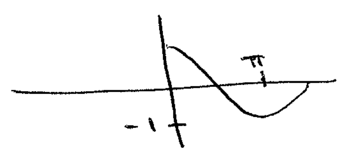
split it up! before reducing

$\dfrac{\sin\alpha\sin\beta}{\sin\alpha\sin\beta} \to 1$

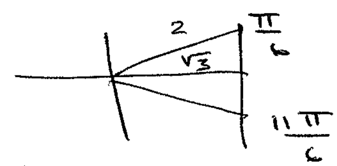
$= \cot\alpha\cot\beta + 1$ ✓

#7. Evaluate

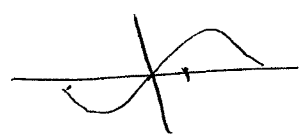
$\cos(\pi) = -1$

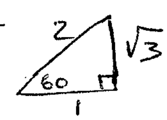



$\cos\left(\dfrac{11\pi}{6}\right) = \cos\dfrac{\pi}{6} = \dfrac{\sqrt{3}}{2}$

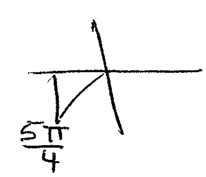



$\sin\left(\dfrac{\pi}{3}\right) = \dfrac{\sqrt{3}}{2}$

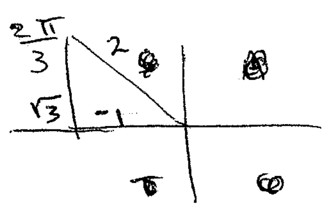





$\sin\left(\dfrac{5\pi}{4}\right) = -\sin\dfrac{\pi}{4} = -\dfrac{\sqrt{2}}{2}$




$\tan\left(\dfrac{2\pi}{3}\right) = -\sqrt{3}$

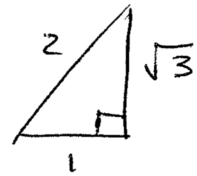


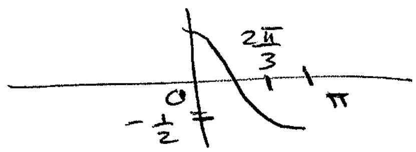

$\sin\left(\dfrac{5\pi}{12}\right) = \sin\left(\dfrac{3\pi}{12}+\dfrac{2\pi}{12}\right) = \sin\dfrac{\pi}{4}\cos\dfrac{\pi}{6} + \cos\dfrac{\pi}{4}\sin\dfrac{\pi}{6}$

#7 con'd

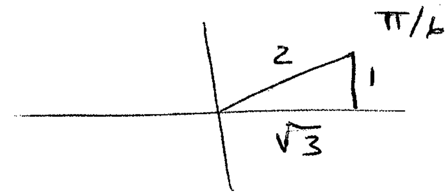
$$= \frac{\sqrt{2}}{2}\cdot\frac{\sqrt{3}}{2} + \frac{\sqrt{2}}{2}\cdot\frac{1}{2} = \frac{\sqrt{6}+\sqrt{2}}{4}$$

#8 - $\arccos\left(-\frac{1}{2}\right) = \frac{2\pi}{3}$



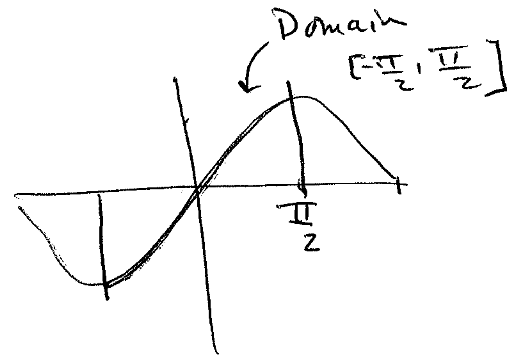




Dom for $\cos^{-1} x$ is $[0, \pi]$




- $\tan^{-1}\left(\frac{\sqrt{3}}{3}\right) = \tan^{-1}\left(\frac{1}{\sqrt{3}}\right)$

$= \pi/6$

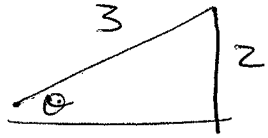



- $\arcsin\left(\sin\left(\frac{5\pi}{4}\right)\right) = -\frac{\pi}{4}$

$\downarrow$

Not in dom of sin for arc sin to be defined

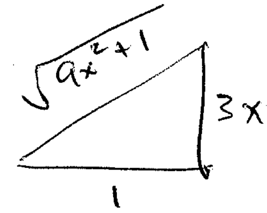
- $\cos\left(\sin^{-1}\left(\frac{2}{3}\right)\right)$




$\sqrt{9-4} = \sqrt{5}$

$\cos\theta = \sqrt{5}/3$

- $\sec(\arctan(3x))$ make the triangle



#8 con'd

So, $\sec(\arctan(3x)) = \sqrt{9x^2+1}$

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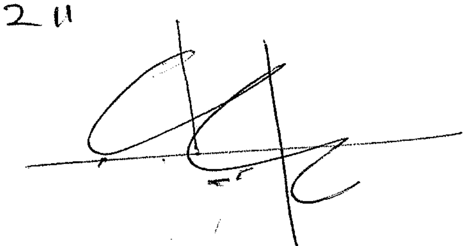
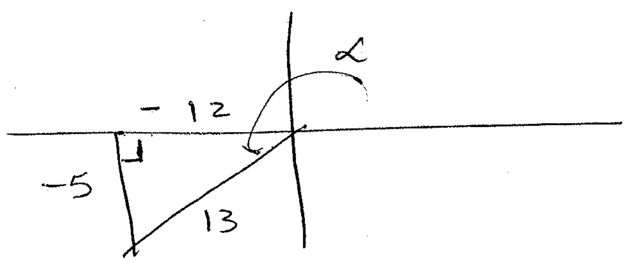
#14) $\sin\alpha = -\frac{5}{13}$ $\cos\beta = \frac{4}{5}$

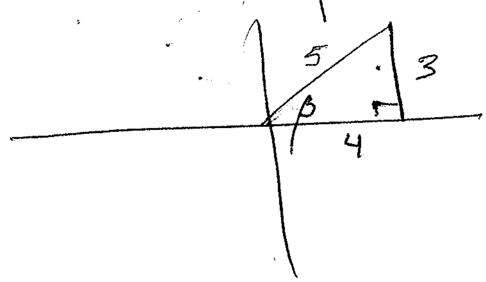
such $\frac{3\pi}{2} < \alpha < 2\pi$

$\frac{3\pi}{2} < \beta < 2\pi$



Draw the triangle

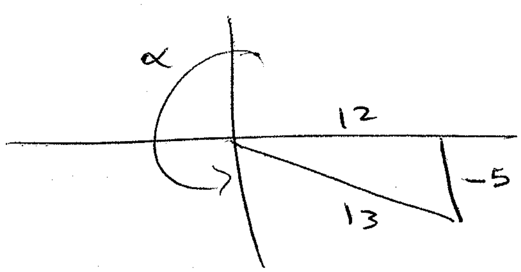


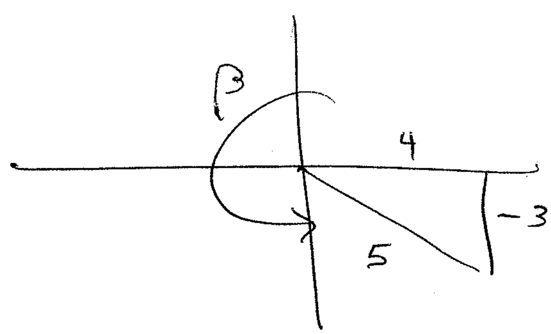




~~sin β = 3/5~~

But $\frac{3\pi}{2} < \alpha < 2\pi$ + $\frac{3\pi}{2} < \beta < 2\pi$

so the quadrants are both $\underline{\text{IV}}$



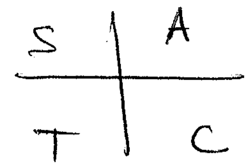
# 14 con'd

a) $\sin\beta = -3/5$ b) $\cos\alpha = \cancel{12/5}\ \frac{12}{13}$

c) $\cos(\alpha+\beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta$

$$= \underset{\text{found}}{\frac{12}{13}} \cdot \underset{\text{given}}{\frac{4}{5}} - \underset{\text{given}}{\frac{-5}{13}} \cdot \underset{\text{found}}{\frac{-3}{5}}$$

$$= \frac{48}{65} - \frac{15}{65} = \frac{33}{65} > 0$$




d) $\alpha+\beta$ is either in I or IV

since $\cos(\alpha+\beta) > 0$

But $\frac{3}{2}\pi < \alpha < 2\pi$ + $\frac{3\pi}{2} < \beta < 2\pi$

so $\frac{3\pi}{2} + \frac{3}{2}\pi < \alpha+\beta < 2\pi + 2\pi$

$3\pi < \alpha+\beta < 4\pi$ hence Q IV

e) $\sin(2\alpha) = 2\sin\alpha\cos\alpha = 2\left(-\frac{5}{13}\right)\left(\frac{12}{13}\right) = \frac{-120}{169}$
(identity)

f) $2\alpha$ is in Q IV since $\frac{3}{2}\pi < \alpha < 2\pi$

hence $3\pi < 2\alpha < 4\pi$

g) $\cos\left(\frac{\beta}{2}\right) = -\sqrt{\frac{1+\cos\beta}{2}} = -\sqrt{\frac{1+4/5}{2}} = -\sqrt{9/10}$

formula given

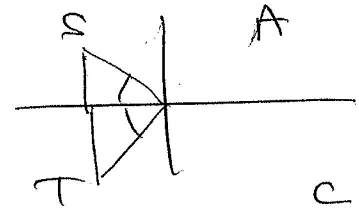
h) $\frac{\beta}{2}$ is in Q II b/c

$\frac{3\pi}{2} < \beta < 2\pi$ so dividing by 2 gives

$\frac{3\pi}{4} < \frac{\beta}{2} < \pi$

(Just like technique in (d) + (f) )

12. $2\cos(5x) = -\sqrt{2}$




$$\cos(5x) = -\frac{\sqrt{2}}{2}$$

$5x$ is either $\frac{3}{4}\pi$ or $\frac{5\pi}{4}$

$$5x = \frac{3}{4}\pi \longrightarrow x = \frac{3\pi}{20} \text{ or } \frac{5\pi}{20} = \frac{\pi}{4}$$

But we want all solns, so add ~~2π~~ $2\pi n$ to each answer, meaning going around the circle CW or CCW $n$ times.

$$\theta = 5x = \frac{3}{4}\pi + 2\pi n$$

$$\boxed{x = \frac{3}{20}\pi + \frac{2\pi}{5}n}$$

$$\theta = 5x = \frac{5\pi}{4} + 2\pi n$$

$$\boxed{x = \frac{\pi}{4} + \frac{2\pi}{5}n}$$

~~and~~ $\cancel{x = \frac{3\pi}{4} + \frac{2}{5}n}$

13. To solve a trig eqn, isolate an expression in trig($x$), set = zero.

This is done by factoring, same as w̄ polynomial.

$$1 + \sin x = 2\cos^2 x \quad \text{on } [0, 2\pi)$$

We need to change it all to $\sin x$ or $\cos x$.

$$1 + \sin x = 2\cos^2 x \longrightarrow$$ Think of mother identity $\underline{\sin^2 x} + \cos^2 x = 1$

$$1 + \sin x = 2(1 - \sin^2 x)$$

$$1 + \sin x = 2 - 2\sin^2 x$$

$$2\sin^2 x + \sin x - 1 = 0$$

$$(2\sin x - 1)(\sin x + 1) = 0$$

$$2\sin x - 1 = 0 \longrightarrow \sin x = 1/2 \rightarrow x = \pi/6 \ \& \ 5\pi/6$$

$$\sin x + 1 = 0 \longrightarrow \sin x = -1 \rightarrow x = 3\pi/2$$