Math 108 - Practice Test 4 - Trigonometry Unit

1. Name **3 angles coterminal** with $\theta = \frac{\pi}{3}$. Include a negative angle. Express each in radian and degree measure.

2. Find the length of the arc cut by an angle of 150°. for a circle of radius 2 ft.

3. Find the values of the other 5 trig fcns. for a right triangle where $\tan \alpha = 5$.

4. The terminal side of angle θ goes through the pt (2, -5). What is the pt. on the unit circle where the terminal side intersects it?

5. An angle α in std position intersects a pt where $\cos \alpha = -3/5$.
   a) What is the x-coordinate of the pt. whose y-coordinate is -8?

b) At what pt. does the terminal side intersect the unit circle for the same angle α?

6. Without a calculator or table, evaluate $\sin^2 10^\circ + \sin^2 80^\circ$.
(Hint: See Sec 8.3 #7 soln.)

7. In which quadrant is the terminal side of the angle θ?

a) $\tan\theta = 7$, $\cos\theta < 0$

b) $\sin\theta > 0$, $\sec\theta = 4$

8. a) True or false: $\cos\left(\frac{\pi}{2} - x\right) = \cos\left(x - \frac{\pi}{2}\right)$

b) True or false: $\sin\left(\frac{\pi}{2} - x\right) = \sin\left(x - \frac{\pi}{2}\right)$

c) True or false: $\cos x = \sin\left(x + \frac{\pi}{2}\right)$

9. Sketch the graph of the following (2 periods):

a) $g(x) = \cos\left(x + \frac{\pi}{3}\right) + 1$

b) $g(x) = -\frac{1}{2}\tan\left(x - \frac{\pi}{2}\right)$

10. Find an equation for a graph described as a sine function with amplitude $\frac{1}{2}$, period 3, shifted 1 unit down.

11.) Find the domain for each fcn:

Example Find dom $f(x) = \sin^{-1}(1-x)$.

Since range of $\sin\theta$ is $[-1, 1]$ on $[-\frac{\pi}{2}, \frac{\pi}{2}]$

then, the domain of $\sin^{-1}(1-x)$ is gotten by

$$-1 \leq 1-x \leq 1$$

$$-2 \leq -x \leq 0$$

$$2 \geq x \geq 0 \quad \text{or} \quad [0, 2]$$

a) Find dom $f(x) = \cos^{-1}(x+3)$

b) Find dom $f(x) = \arctan(x^2 + x)$

15. Verify the identities:

a) $\dfrac{\sin x}{1+\cos x} + \cot x = \csc x$

b) $\dfrac{1-\cos x}{\cos x} = \dfrac{\tan x}{1+\sec x}$

16. Use sum + diff formulas to evaluate:

a) $\cos^2\left(\frac{\pi}{8}\right) - \sin^2\left(\frac{\pi}{8}\right)$

b) $\tan\left(\frac{\pi}{5} - \frac{\pi}{30}\right)$

c) $\sin\left(\frac{5\pi}{12}\right)$

d) $\cos\left(22\frac{1}{2}^\circ\right)$

12. Evaluate w/o calculator:

a) $\sin^{-1}(-\sqrt{2}/2)$

b) $\cos^{-1}(\cos(5\pi/4))$

c) $\tan^{-1}(\tan \pi/6)$

d) $\sec\left(\cos^{-1}\left(\frac{1}{2}\right) - \sin^{-1}\left(\frac{\sqrt{3}}{2}\right)\right)$

e) $\cos(\tan^{-1} 7)$

13. Find x that makes the statement true (if any):

a) $\sin^{-1}\left(\frac{3}{5}\right) = \cos^{-1} x$

b) $\tan^{-1} 1 = \sin^{-1} x$

c) $\cos^{-1}(-1) = \sin^{-1} x$

14. Write each as an expression in terms of x with no trig fcn:

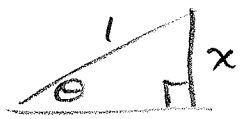
a) $\cos(\tan^{-1}(x+1))$

b) $\sin(\arccos(2x))$

17. Look at #10 in Sec 8.8.

Ex Express $\sin(2\sin^{-1}(x))$ as a fcn. of x with no trig fcn.

Solution • Think of $\sin(2\sin^{-1}(x))$ as $\sin(2\theta)$ where $\theta = \sin^{-1}(x)$

• Draw the △: 

$\sin\theta = \frac{x}{1}$

• Find missing side: $\sqrt{1^2 - x^2} = \sqrt{1-x^2}$

• Use double angle formula:

$\sin 2\theta = 2\sin\theta\cos\theta$

• Refer to the △ for $\sin\theta$, $\cos\theta$

$\sin 2\theta = 2 \cdot \frac{x}{1} \cdot \frac{\sqrt{1-x^2}}{1} = 2x\sqrt{1-x^2}$ //

Now do this:

Write $\sin(2\cos^{-1}(x+1))$ in terms of x w/o trig fcn.

18. Replace ? with correct expression:

a) $\sin(?) = \pm\sqrt{\frac{1-\cos 40}{2}}$

b) $\cos^2(4x) = \pm\sqrt{\frac{1-?}{2}}$

19. Evaluate w/o calculator:

a) $\sin\left(\frac{\pi}{8}\right)$

b) $\cos(195°)$

19. Express $\sin^4 x$ in terms of single-powered trig fcns. (See Ex 8.8.13 p. 276)

20. Solve for x on $[0, 2\pi)$

a) $-\cot(2x) = 2 + \tan(2x)$

b) $2\sin^2 x + \sin x - 1 = 0$