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p. 392 Work problems

These problems are solved by setting up rational equations of the type

$$\boxed{\frac{1}{x} + \frac{1}{y} = \frac{1}{t}}$$

where x = time for Worker A to do a job

y = time for Worker B to do the same job

t = time for the two workers to do the job if they work together

The equation shows the sum of the portion of the job done by each worker to give the portion completed if they work together. For example, if I do $\frac{1}{2}$ the job in an hour and Richard does $\frac{1}{3}$ the job in an hour, together we do

$$\frac{1}{2} + \frac{1}{3} = \frac{3}{6} + \frac{2}{6} = \frac{5}{6} \text{ job in an hour.}$$

Not all problems are this straightforward, but the pattern is essentially the same.

#32 x = time for snow machine to produce enough snow to open a resort

y = " " enough snow to fall to open resort

$$\frac{1}{x} + \frac{1}{y} = \frac{1}{t} \leftarrow \text{Find } t\text{, time that snowfall + manufacture of snow together produce enough snow.}$$

$x = 12h$

$y = 36h$

$$\frac{1}{12} + \frac{1}{36} = \frac{1}{t} \xrightarrow[\text{LCD} = 36t]{} \frac{3t + t}{36t} = \frac{36}{36t} \longrightarrow$$

(2)

#32 (con'd) $4t = 36 \rightarrow \boxed{t = 9 \text{ hr}}$

In #32, we solved a simple problem of 2 machines working together from the beginning till the end of a job.

#37, 38 + 40 are like #32.

#37. x = time for machine 1 = 10 h
y = " " " 2 = 12 h
z = " " " 3 = 15 h

t = time for machines 1, 2, 3 working together

$$\frac{1}{x} + \frac{1}{y} + \frac{1}{z} = \frac{1}{t} \longrightarrow \frac{1}{10} + \frac{1}{12} + \frac{1}{15} = \frac{1}{t}$$

$$LCD = 60t : \quad 60t\left(\frac{1}{10} + \frac{1}{12} + \frac{1}{15}\right) = 60t\left(\frac{1}{t}\right)$$

$$\longrightarrow 6t + 5t + 4t = 60$$

$$15t = 60$$

$$\boxed{t = 4 h}$$

#38) x = time for goat = 12 days
y = " " cow = 15 days
z = " " horse = 20 days
t = time for animals together

$$\frac{1}{x} + \frac{1}{y} + \frac{1}{z} = \frac{1}{t} \longrightarrow \frac{1}{12} + \frac{1}{15} + \frac{1}{20} = \frac{1}{t}$$

$$LCD = 60t: \quad 60t\left(\frac{1}{12} + \frac{1}{15} + \frac{1}{20}\right) = 60t\left(\frac{1}{t}\right)$$

$$\longrightarrow 5t + 4t + 3t = 60$$

$$12t = 6$$

$$\boxed{t = 5 \text{ days}}$$

#40) x = time for Computer 1 = 20 min alone

y = " " " 2 = 30 min alone

z = " " " 3 = 60 min alone

t = time for computers together

$$\frac{1}{x} + \frac{1}{y} + \frac{1}{z} = \frac{1}{t} \rightarrow \frac{1}{20} + \frac{1}{30} + \frac{1}{60} = \frac{1}{t}$$

$$LCD = 60t:$$

$$60t\left(\frac{1}{20} + \frac{1}{30} + \frac{1}{60}\right) = 60t\left(\frac{1}{t}\right)$$

$$\longrightarrow 3t + 2t + t = 60$$

$$6t = 60$$

$$\boxed{t = 10 \text{ min}}$$

For all these problems, we added, in effect, the <u>portion</u> of the job each worker completed in a given unit of time, based on ~~its~~ their individual rates, <u>while working together for the whole job</u>.

For the next problems, the workers don't start + finish together.

We must break the whole job into parts.

The new model looks like this:

$$\left(\text{Rate of worker 1}\right)\left(\text{time on task}_{\text{alone}}\right) + \left(\text{Rates of worker 1} + \text{worker 2}\right)\left(\text{time on task together}\right) = 1$$

(as in one whole job done)

For example, suppose worker 1 does a job in x units of time + worker 2 does the same job in y units of time.

Further suppose worker 1 goes it alone for t_1 min and worker 2 joins him for t_2 min to finish the "whole" job.

Then:

$$\boxed{\frac{1}{x}\cdot t_1 + \left(\frac{1}{x}+\frac{1}{y}\right) t_2 = 1}$$

This is the model for $\boxed{\#\ 33, 34, 35, 36}$.

#33. x = time for bricklayer to do entire job alone

$y = 2x$ = time for apprentice to do same job alone

$t_1 = 10$ h. for apprentice working alone to finish a job that the two worked together on for 6h. That is,

$t_2 = 6$ h

(5)

#33 con'd The "y" is the time of the worker alone so adjust the model

$$\frac{1}{y} \cdot t_1 + \left(\frac{1}{y} + \frac{1}{x}\right) t_2 = 1$$

$y = 2x$

$$\frac{1}{2x} \cdot t_1 + \left(\frac{1}{2x} + \frac{1}{x}\right) t_2 = 1$$

$$\frac{1}{2x} \cdot 10 + \left(\frac{1}{2x} + \frac{1}{x}\right)(6) = 1$$

$$\frac{5}{x} + \frac{3}{x} + \frac{6}{x} = 1 \longrightarrow \frac{14}{x} = 1$$

Bricklayer alone $\boxed{x = 14\text{ h}}$

#34 Remember, the models are

$\frac{1}{t} = \frac{1}{x} + \frac{1}{y} = \frac{1}{t}$ when workers work together for entire job.

$1 = \frac{1}{x} \cdot t_1 + \left(\frac{1}{x} + \frac{1}{y}\right) t_2$ when workers work together for part of the job.

$x = 8$ h for roofer to do entire job

y = time for apprentice to do entire job (unknown)

$t_1 = 10$ h for apprentice working alone to finish job

$t_2 = 2$ h that apprentice + roofer worked together

$$1 = \frac{1}{y} \cdot t_1 + \left(\frac{1}{x} + \frac{1}{y}\right) t_2$$

$$1 = \frac{1}{y} \cdot 10 + \left(\frac{1}{8} + \frac{1}{y}\right) \cdot 2$$

(6)

$$1 = \frac{10}{y} + \frac{2}{8} + \frac{2}{y} = \frac{12}{y} + \frac{2}{8}$$

$LCD = 8y$: $\quad 8y(1) = \left(\frac{12}{y} + \frac{2}{8}\right) 8y$

$$8y = (12)(8) + 2y$$

$$6y = 96 \longrightarrow \boxed{y = 16\text{ h}}$$ apprentice to do job alone

(#35) Again, a machine works alone, and with another machine.

$x = 40$ min = time for machine 1 to fill entire order alone

y = time for machine 2 to fill entire order alone (unknown)

t_1 = time Machine 1 worked alone = 15 min

t_2 = time mach 1 + mach 2 worked together = 15 min

$$1 = \frac{1}{x} \cdot t_1 + \left(\frac{1}{x} + \frac{1}{y}\right) t_2$$

$$1 = \frac{1}{40} \cdot 15 + \left(\frac{1}{40} + \frac{1}{y}\right) 15$$

$$1 = \frac{15}{40} + \frac{15}{40} + \frac{15}{y} \qquad LCD = 40y$$

$$40y \cdot 1 = \left(\frac{30}{40} + \frac{15}{y}\right) 40y$$

$$40y = 30y + 600$$

$$10y = 600 \longrightarrow \boxed{y = 60\text{ min}}$$

for mach 2 to do ~~jobs~~ entire job alone

(7) One last type of work problem is that in which work is <u>undone</u> partially while it's being done. This is what a pipe filling + another emptying a tank represents.

(#39) An inlet pipe fills a water tank in 45 min. The outlet pipe empties it in 30 min.

How long will it take to empty a full ~~the~~ tank with both pipes open?

Now the "whole job" is to get an empty tank. The pipe <u>in</u> delivers water more slowly than the pipe <u>out</u> dumps the water. The time it takes to empty the tank is found from the difference in the rates of filling + emptying:

Let x = time to empty tank = 30 min
y = " " fill " = 45 min
t = time together to empty tank.

$\frac{1}{x} - \frac{1}{y}$ is the portion of the tank emptied in 1 min. (that is, the rate at which the tank empties out).

$$\frac{1}{x} - \frac{1}{y} = \frac{1}{t} \longrightarrow \frac{1}{30} - \frac{1}{45} = \frac{1}{t}$$

$LCD = 90t$: $3t - 2t = 90 \rightarrow \boxed{t = 90 \text{ min}}$

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Take home problems #41, 42, e4

One last example

When the job to be done is not 1 whole but a number of items, like in problem #41, the model is similar to the first.

$$\frac{1}{x} + \frac{1}{y} = \frac{\text{number of items}}{t}$$

#41 x = time for clown 1 to blow up a balloon = 2 min.

y = time for clown 2 to blow up a balloon = 3 min

z = time between balloons popping

t = time to inflate 76 balloons.

Here, the rates of the clowns are diminished by the balloons popping:

$$\frac{1}{x} + \frac{1}{y} - \frac{1}{z} = \frac{\text{\# of balloons}}{t}$$

$$\frac{1}{2} + \frac{1}{3} - \frac{1}{5} = \frac{76}{t}$$

LCD = $30t$: $15t + 10t - 6t = 76 \cdot 30$

$19t = 2280$

$t = 120$ min to inflate 76 balloons

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Note! If you look at the first model,

$\frac{1}{x} + \frac{1}{y} = \frac{1}{t}$, and multiply through

by t, you get $\frac{1}{x} \cdot t + \frac{1}{y} \cdot t = 1$

This just looks like the second ~~problem~~ model, but with one value of t, namely, time together. We can see the rationale for the second model from this:

$$\boxed{\frac{1}{x} \cdot t + \frac{1}{y} \cdot t = 1 \quad \text{vs.} \quad \frac{1}{x} \cdot t_1 + \left(\frac{1}{x} + \frac{1}{y}\right) t_2 = 1}$$

From here, your task is easy. Choose one of these 2 models, depending on the situation.

#36 x = time it takes small printer to do job alone (unknown)

y = time it takes large printer to do job alone = 30 min

t_1 = time small printer needs to finish job = 40 min

t_2 = time small + large work together = 10 min

$$1 = \frac{1}{x} \cdot t_1 + \left(\frac{1}{x} + \frac{1}{y}\right) t_2$$

$$1 = \frac{1}{x} \cdot 40 + \left(\frac{1}{x} + \frac{1}{30}\right) \cdot 10$$

$$1 = \frac{40}{x} + \frac{10}{x} + \frac{10}{30} = \frac{50}{x} + \frac{1}{3}$$

LCD = $3x$: $\quad 3x = 150 + x \rightarrow \boxed{x = 75 \text{ min for small printer alone}}$