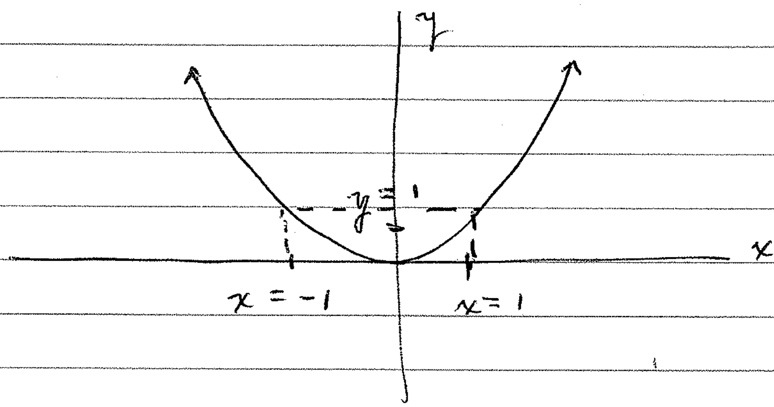
$\boxed{9-24-13}$ Proof that $\sqrt{x^2} = |x|$

If we write ~~[illegible]~~ $y = x^2$ and take the root of both sides, we have $\pm\sqrt{y} = x$. This is clear from the graph of $y = x^2$



But we are accustomed to putting $\pm$ on the x side, that is:

$$y = x^2 \longrightarrow \sqrt{y} = \sqrt{x^2} = \pm x$$

It's really a matter of order in which we evaluate a function for x.

$$\left.\begin{aligned} f(1) &= 1^2 = 1 \\ f(-1) &= (-1)^2 = 1 \end{aligned}\right\} f^{-1}(1) = 1 \text{ or } -1$$

depending on how you restrict the domain of $f(x)$ (since f is not invertible).

We notice also that $y = |x| = \begin{cases} x, & x \geq 0 \\ -x, & x < 0 \end{cases}$

This is the same definition of the rule for $y = \sqrt{x^2}$ and the same domain! Hence $\sqrt{x^2} = |x|$.