Exam #2 Key / Guide

1. (b) $c < 2$

Explanation $f(x) = 2x^2 - 4x + c$ has 2 real roots when when $b^2 - 4ac > 0$. Thus, solving for c:

$$(-4)^2 - 4(2)(c) > 0$$
$$16 > 8c \longrightarrow c < 2$$

(b) reflection through x-axis w/ vertical shift down 2 units

Explanation g(x) is transformation of f(x) s.t.

$$g(x) = \underbrace{-f(x)}_{\substack{\text{reflects } y \text{ to } -y \\ (\text{over } x\text{-axis})}} \underbrace{-2}_{\text{moves down 2}}$$

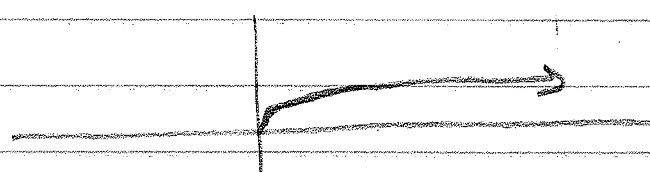
(a) f(x) has no HA

Explanation
$$f(x) = \frac{x^2 - 5x - 6}{x + 2} = \frac{P(x) \rightarrow \text{deg } 2}{Q(x) \rightarrow \text{deg } 1}$$

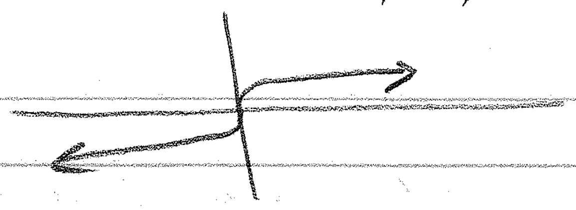
If deg P > deg Q, then $f \rightarrow \infty$ as $x \rightarrow \infty$. This is contrary asymptotic behavior.

(d) $[0, \infty)$ is range

Explan. $f(x) = \sqrt[6]{x}$ even roots are positive or zero



Note - odd roots are +, −, zero



(c) $-2/3 = m$

Explan $2x + 3y = 5 \Rightarrow y = -\frac{2}{3}x + \frac{5}{3}$

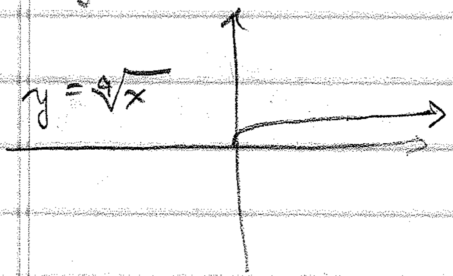
(d) $x=3$ is not even possible root according to rational root thm.

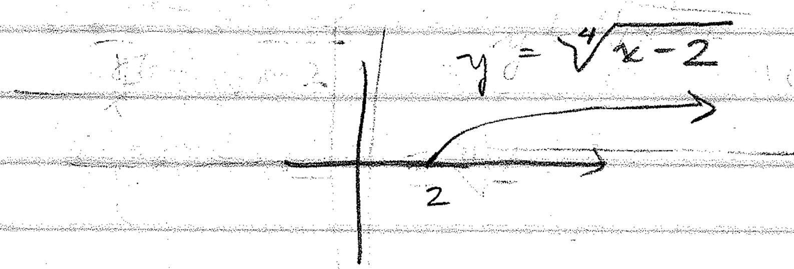
Explan. The possible roots are $\{p/q\}$ where p is factor of a_0 and q a factor of a_n in $f(x) = a_n x^n + \dots + a_1 x + a_0$

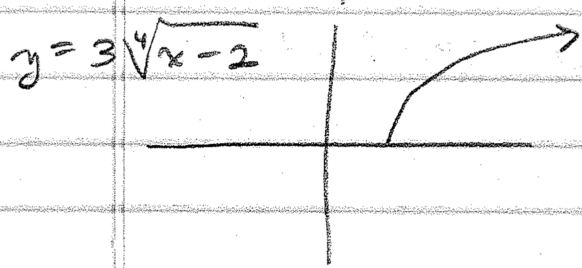
$f(x) = 3x^8 + \dots - 11$ so possible roots are $\left\{\pm \frac{11}{3}, \pm 11, \pm \frac{1}{3}\right\}$

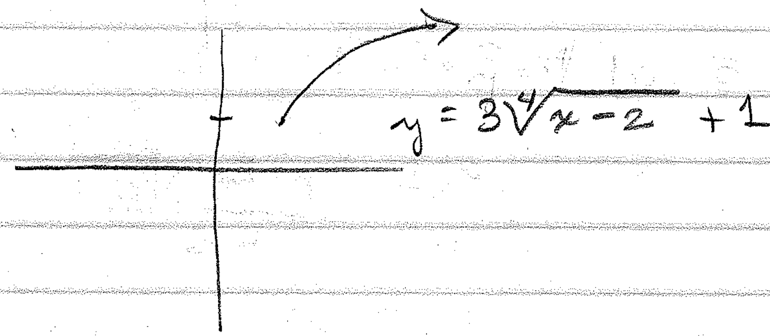
2, Sketch the steps of the transformation from

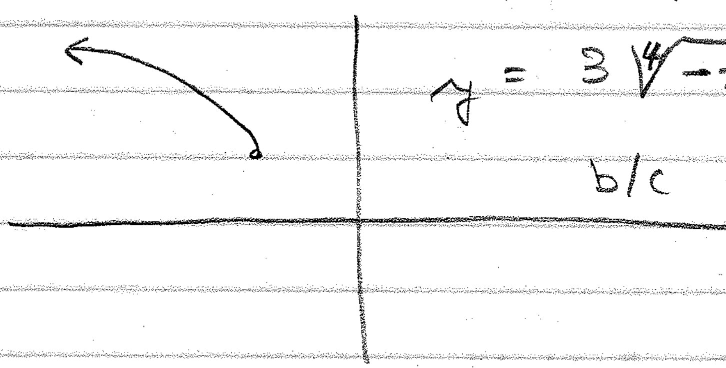
$y = \sqrt[4]{x}$ to $g(x) = 3\sqrt[4]{-x-2} + 1$











$y = 3\sqrt[4]{-x-2} + 1$

b/c $x \rightarrow -x$ by reflection over y-axis

#3 See graph

#4 $A = (1, -7)$ $B = (4, 5)$

Use pt-pt form or find slope, then b.

$m = \frac{5 - -7}{4 - 1} = \frac{12}{3} = \boxed{4 = m}$

$y = 4x + b$ $\qquad y = 4x - 11$

for $(1, -7)$ $\quad -7 = 4(1) + b$

$\boxed{b = -11}$

$$\text{Midpt} = \left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right) = \left(\frac{1+4}{2}, \frac{-7+5}{2}\right)$$

$$= \left(\frac{5}{2}, -1\right)$$

$$d = \sqrt{(x_1-x_2)^2+(y_1-y_2)^2}$$

$$= \sqrt{(4-1)^2+(5-(-7))^2} = \sqrt{9+144} = \sqrt{153}$$

5. a) $-3x^2-7x+5=0$ Use quad. form.

$$x = \frac{7 \pm \sqrt{109}}{-6}$$

b) $x^4-7x^2=-12$

$$x^4-7x^2+12=0$$

$$(x^2-4)(x^2-3)=0 \qquad x = \pm 2, \pm\sqrt{3}$$

c) $(12x+5)^2 = 11$

$$12x+5 = \pm\sqrt{11}$$

$$x = \frac{-5 \pm \sqrt{11}}{12}$$

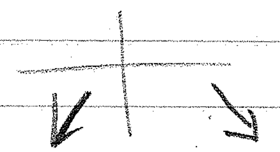
#6 $g(x) = -x^4 + 5x^3 - 2x^2 - 20x + 24$

$x = 2, -2$ are two of the roots

a) deg $= 4$

b) lead coef. is -1

c)



since it's even w/ neg l.c.

d) $g(0) = 24$, so $(0, 24)$ is y-int

e) Since $x = 2, -2$ are roots, then $x^2 - 4$ is a factor. By long $\div$

$$
\begin{array}{r}
-x^2 + 5x - 6 \\
x^2 - 4 \enclose{longdiv}{-x^4 + 5x^3 - 2x^2 - 20x + 24} \\
-(-x^4 \qquad\quad + 4x^2) \qquad\qquad \\
5x^3 - 6x^2 - 20x \qquad \\
-(5x^3 \qquad\quad - 20x) \qquad \\
-6x^2 \qquad + 24 \\
-(-6x^2 \qquad + 24) \\
0
\end{array}
$$

$\therefore g = (x+2)(x-2)(-x^2 + 5x - 6)$

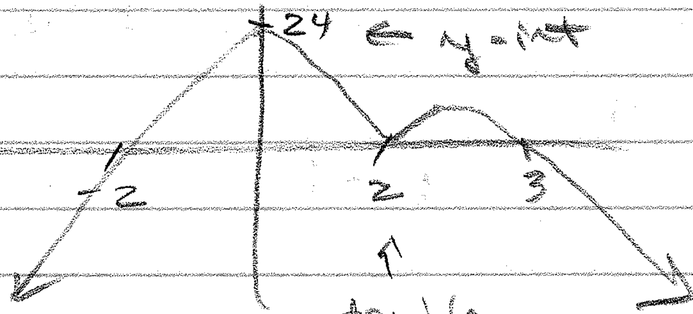
$= -(x+2)(x-2)(x^2 - 5x + 6)$

$= -(x+2)(x-2)(x-3)(x-2)$

f) x-ints (roots): $\pm 2, 3, 2$ in coordinate form $(2,0), (-2,0), (3,0), (2,0)$

double root

g) Sketch as roughly



#7 See graph

#8 a) $f(x) = \dfrac{2x^2 + x - 5}{x^2}$ deg P = deg Q

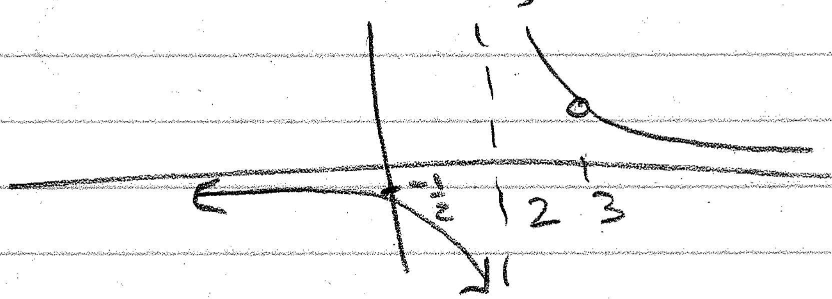
HA is $y = \dfrac{2}{1} = 2$ (ratio of l.c.)

VA is $x = 0$ from denom

b) $g(x) = \dfrac{x-3}{x^2 - x + 6} = \dfrac{\cancel{(x-3)}}{\cancel{(x-3)}(x-2)} = \dfrac{1}{x-2}$

Dom: $x \neq 3, 2$

There's a hole at $x = 3$, $y = 1$ $(g(3) = 1)$
and VA of $x = 2$, a HA of $y = 0$



Note
$g(0) = \dfrac{-3}{6} = -\dfrac{1}{2}$

Test 2 Asymptotes of rational fcns

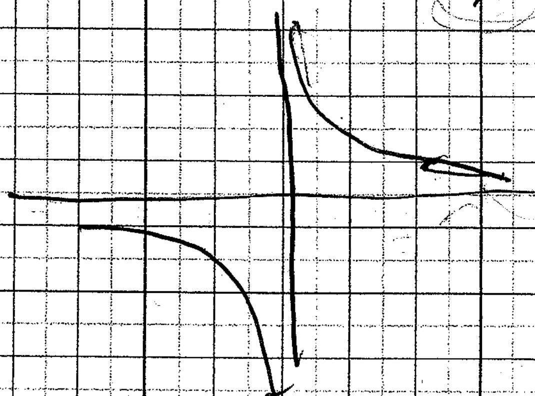
These are determined by comparing deg $N(x)$ to $D(x)$

given $R(x) = \frac{N(x)}{D(x)} = \frac{P(x)}{Q(x)}$

Case A $\deg Q(x) > \deg P(x)$

ex $R(x) = \frac{1}{x}$

as $x \to \infty$
$R(x) \to 0$
as $x \to -\infty$
$R(x) \to 0$



$y = 0$ is HA

Case B $\deg P(x) = \deg Q(x)$

then HA is y = ratio of lead. coeffs

ex $R(x) = \frac{8x^2 - 4x + 3}{2x^2 - 6x + 1}$

$y = \frac{8}{2} = 4$ is HA

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Test #2 Polynomials

#7 Poly of degree n has at **most** n real roots

Since we're given $x=3$, $x=0$, $x=-4$ are the **only** roots

(we look for least deg possible, as asked)

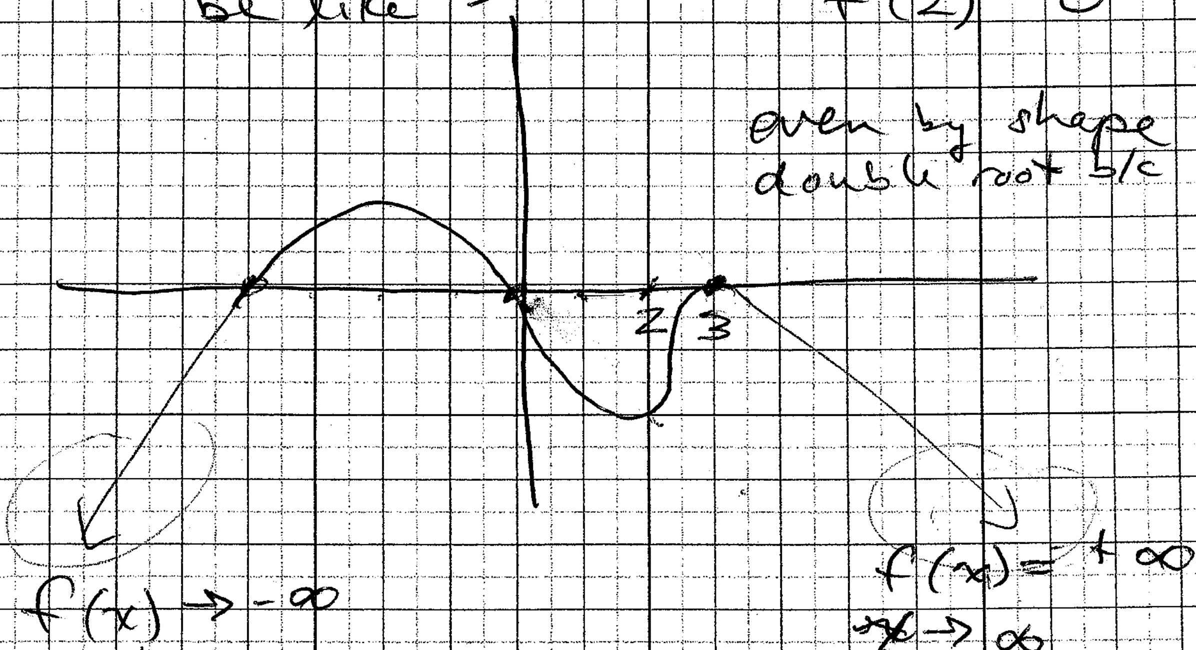
$$P(x) = (x-3)^2(x+4)x = 0$$

or $P(x) = 6(x-3)^2(x+4)x$

Could be like

$f(2) < 0$

even by shape
double root b/c deg 4



$f(x) \to -\infty$
$x \to -\infty$

$f(x) = +\infty$
$x \to \infty$

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#9) $f(x) = \dfrac{x(x-6)\cancel{(x+1)}}{(x-2)\cancel{(x+1)}}$ (factored)

a) Dom: $\{x \mid x \neq 2, -1\}$

b) $f(0) = \dfrac{0(0-6)(0+1)}{(0-2)(0+1)} = 0$ $\boxed{(0,0)}$

c) $f(x) = 0$ when numerator $= 0$

$f(x) = x(x-6) = 0$, $x = 0, 6$

$\boxed{(0,0), (6,0)}$

d) hole at $x = -1$ (see cancelled factor)

$g(-1) = \dfrac{-1(-1-6)}{(-1-2)} = \dfrac{7}{-3} = \dfrac{-7}{3}$

$\boxed{(-1, -7/3)}$

e) $x = 2$ VA

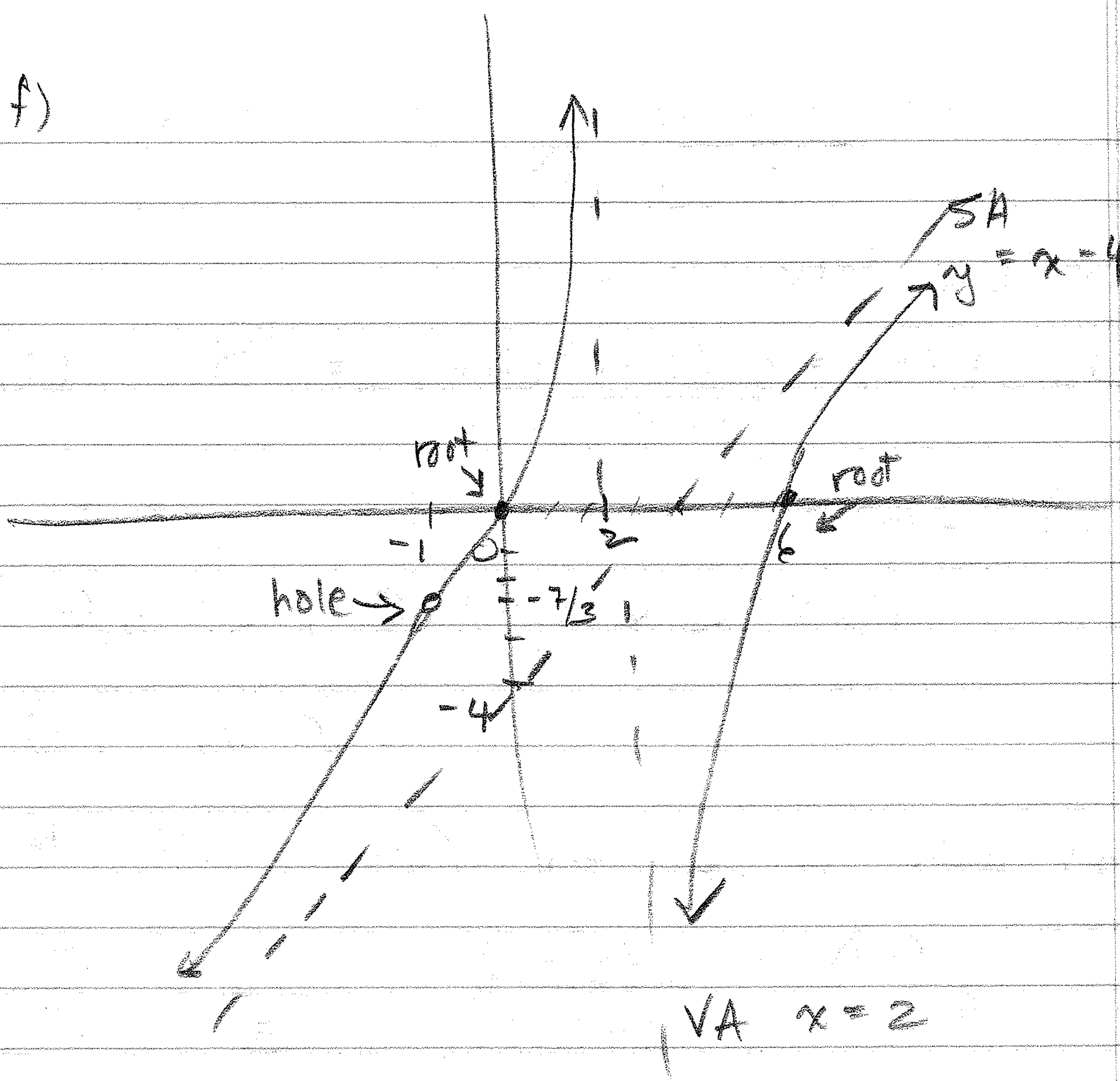
$y = ?$ SA (deg P = deg Q + 1)

$f(x) = \dfrac{x(x-6)}{x-2} = \dfrac{x^2-6x}{x-2} =$

$$\begin{array}{r|l} & x - 4 \\ \hline x-2 & x^2 - 6x \\ & -(x^2 - 2x) \\ \hline & -4x \\ & -(-4x + 8) \\ \hline & -8 \end{array}$$

$\boxed{\text{SA } y = x - 4}$

f)



Test #3 Key + Guide

1. $\begin{cases} 5x - 2y = 1 \\ y + 3 = -x \end{cases}$ Use substitution to "solve" the system

$5x - 2\underbrace{(-x-3)}_{y} = 1 \rightarrow 5x + 2x + 6 = 1$

$\rightarrow 7x = -5$

$\rightarrow \boxed{x = -5/7}$

$y + 3 = -x$

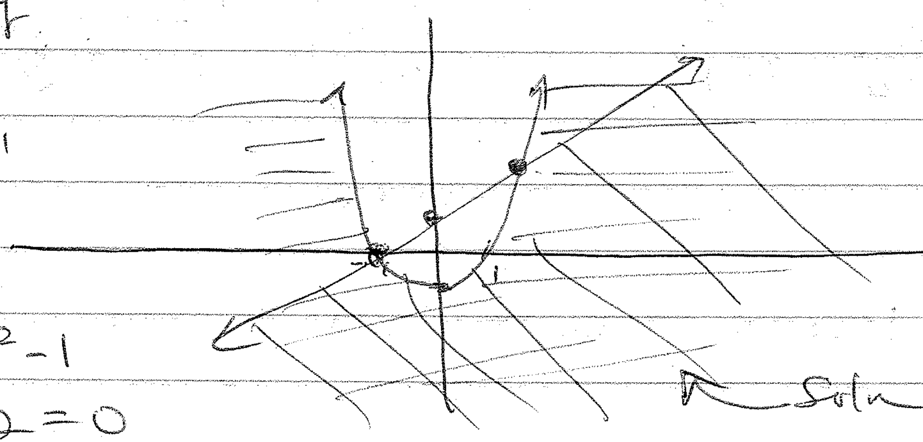
$y + 3 = -(-5/7)$

$y = -16/7 \longrightarrow (-5/7, -16/7)$

2. $\begin{cases} y \le x^2 - 1 \\ -x + y \le 1 \end{cases}$ Solve the system graphically + find vertices algebraically

$\downarrow$

$y \le x + 1$



Solve $x + 1 = x^2 - 1$

$x^2 - x - 2 = 0$

$(x-2)(x+1) = 0$

Vertices at $x = 2, -1 \Rightarrow y = 2^2 - 1 = 3,\ (-1)^2 - 1 = 0$

$(2,3)$ + $(-1,0)$

~~#3) a)~~ $\cancel{\sum_{i=0}^{4}(-1)^i(2i+1) = (-1)^0(2\cdot 0+1) + \sum_{i=1}^{4}(-1)^i(2i+1)}$

$$\cancel{=17}$$

#3 a) Alternating series – have to expand from $i=0$ to 4

$$\sum_{i=0}^{4}(-1)^i(2i+1) = (-1)^0((2\cdot 0)+1) + (-1)^1((2\cdot 1)+1)$$
$$+ (-1)^2((2\cdot 2)+1) + (-1)^3((2\cdot 3)+1)$$
$$+ (-1)^4((2\cdot 4)+1)$$
$$= 1-3+5-7+9 = \boxed{5}$$

b) Rewrite as a sum from $i=6$

$$\sum_{i=4}^{12}\frac{3i}{1-i^2} = \sum_{i=6}^{14}\frac{3(i-2)}{1-(i-2)^2}$$

subtract 2

add 2

c) Use formulas to get the sum in terms of n

$$\sum (i+2)(i-5) = \sum_{i=1}^{n}(i^2-3i-10)$$
$$= \sum_{i=1}^{n} i^2 - 3\sum_{i=1}^{n} i - \sum_{i=1}^{n} 10$$
$$= \frac{n(n+1)(2n+1)}{6} - \frac{3(n)(n+1)}{2} - 10n$$ ← Final

3d) Evaluate $\sum_{i=11}^{30} 4i$

$$\sum_{i=11}^{30} 4i = 4\sum_{i=11}^{30} i = 4\sum_{i=1}^{20} (i+10)$$

add 10

subtract 10

$$= 4\sum_{i=1}^{20} i + 4\sum_{i=1}^{20} 10 = 4\left(\frac{20(21)}{2} + 20\cdot 10\right)$$

$$= \boxed{1640}$$

Alternately

$$\sum_{i=11}^{30} 4i = \sum_{i=1}^{30} 4i - \sum_{i=1}^{10} 4i$$

#4 a) Solve for x

$$|x^2 - 10| = -3x$$

Case 1 | Case 2

When $x^2 - 10 \geq 0$

$\cancel{x^2 - 10 \geq -3x}$

$x^2 - 10 = -3x$ (given)

$x^2 + 3x - 10 = 0$

$\boxed{x = -5,}$ $\cancel{2}$ reject upon checking

When $x^2 - 10 < 0$

$\cancel{x^2 - 10 \leq -(-3x)}$

$-(x^2 - 10) = -3x$

$x^2 - 10 = 3x$

$x^2 - 3x - 10 = 0$

$x = \cancel{5}$, $\boxed{-2} = x$

reject upon checking

~~#4b)~~

#5) a) $|5 - 3x| \geq 8 \implies$ ($5 - 3x \geq 8$ or $5 - 3x \leq -8$) 2 cases

$5 - 3x \geq 8$

$x \leq -1$

$5 - 3x \leq -8$

$x \geq 13/3$

$$\boxed{(-\infty, -1] \cup [13/3, \infty)}$$

Check at $x = 0$

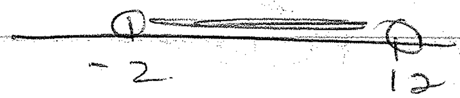
$|5 - 3 \cdot 0| \not\geq 8$

so the intervals found look right

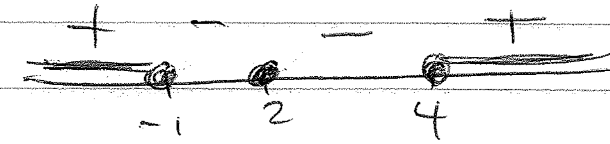
#5b) $|x-5| < 7 \longrightarrow -7 < x-5 < 7$

$-2 < x < 12$

$\boxed{(-2, 12)}$



#5c) $(x-2)^2 (x-4)(x+1) \geq 0$



$\boxed{(-\infty, -1] \cup \{2\} \cup [4, \infty)}$

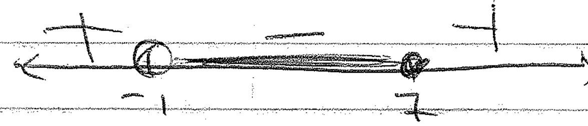
Check intervals for sign + or −
(we want ≥ 0, so +)

d) $\dfrac{3x-5}{x+1} \leq 2$

Look at zeroes of num. + denom.

$\dfrac{3x-5}{x+1} - 2 \leq 0 \longrightarrow \dfrac{3x-5-2x-2}{x+1} \leq 0$

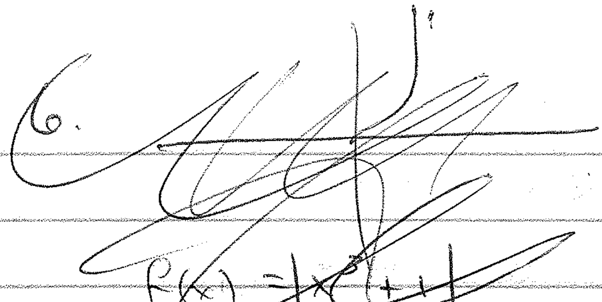
$\longrightarrow \dfrac{x-7}{x+1} \leq 0$



not in domain

$\boxed{(-1, 7]}$

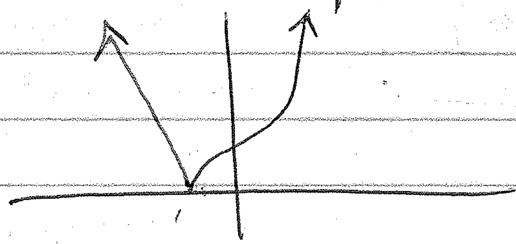
6.



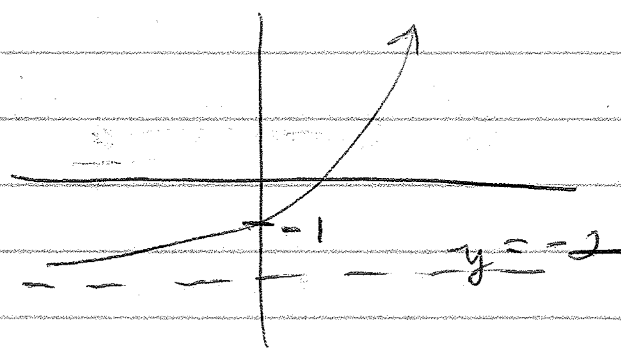
$\cancel{f(x) = |x^3+1|}$

6. <u>Sketches</u>

$f(x) = |x^3+1|$ Take $y = x^3+1$ + flip the negative part



$f(x) = 2^x - 3$ Drop 2^x down 3



$f(x) = \ln x$