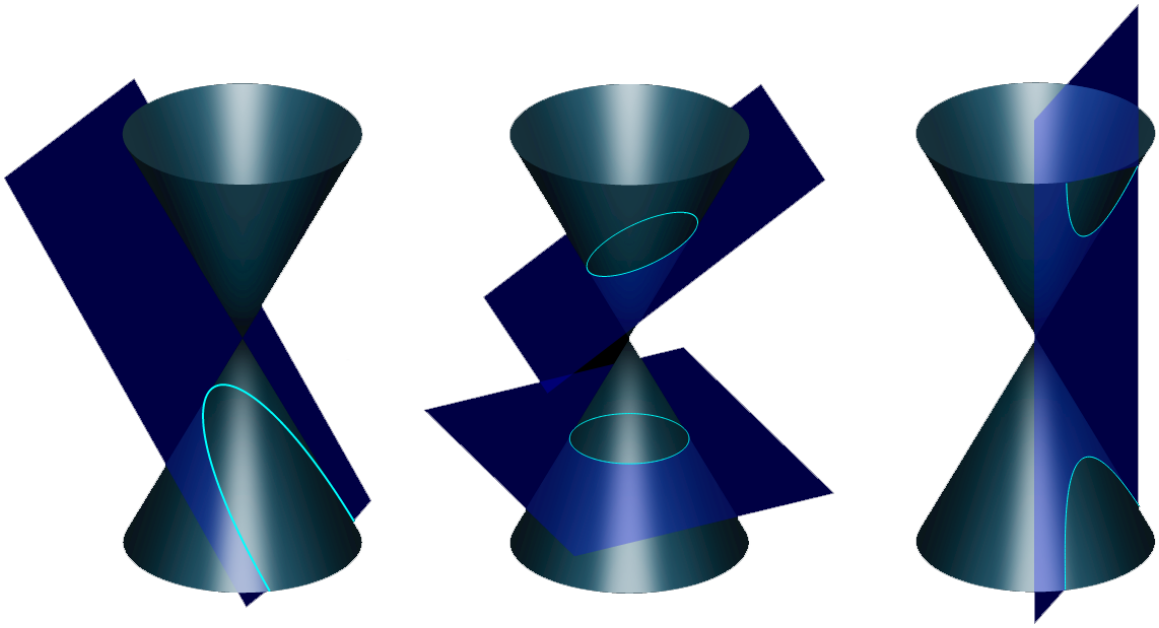
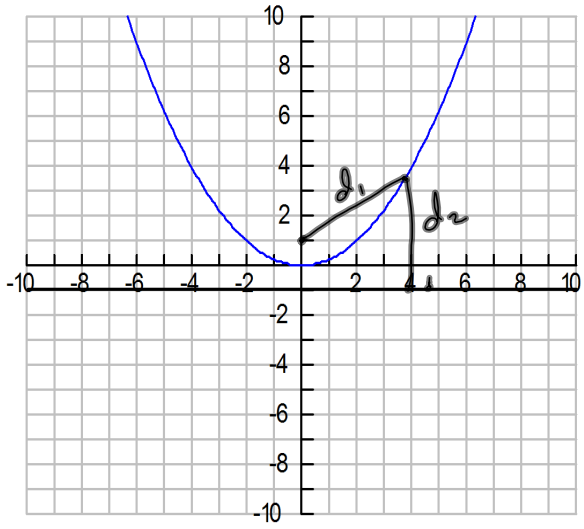
8.1 Conic Sections and Parabolas



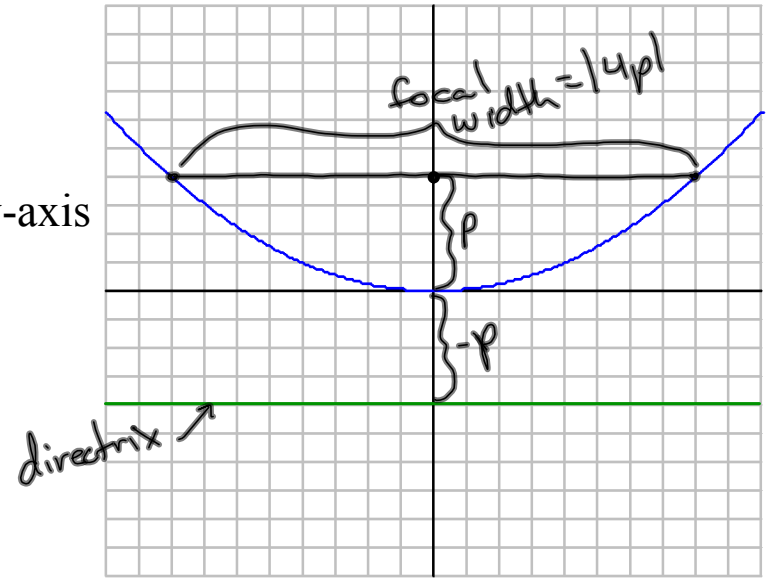
Defn: Parabola - a set of all points in a plane equidistant from a particular line (the directrix) and a particular point (the focus) in the plane.

$d_1 = d_2$



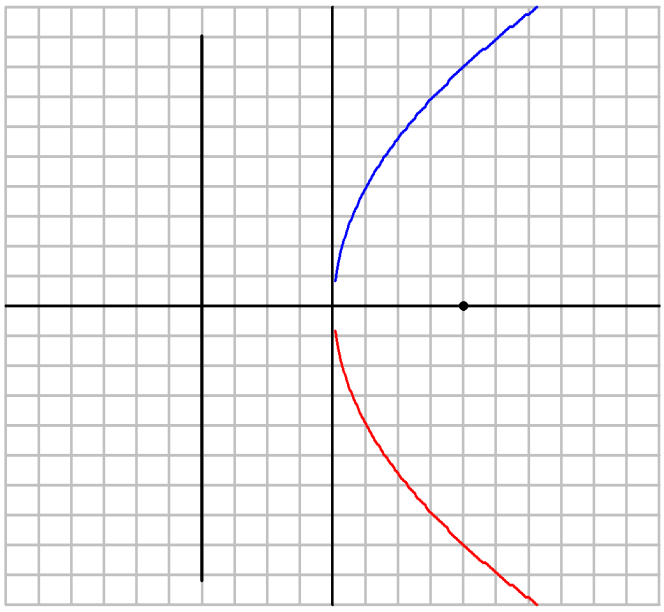
$x^2 = 4py$

1. Opens up or down
2. focus (0,p)
3. Directrix y = -p
4. Axis of symmetry: y-axis
5. Focal Length: p
6. Focal width: |4p|



$y^2 = 4px$

1. Opens Right or left
2. Focus (p,0)
3. Directrix: x = -p
4. Axis of symmetry: x-axis
5. Focal Length: p
6. Focal width |4p|

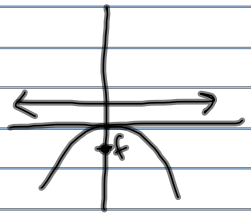


Example 1: Find the focus, the directrix, and the focal width of the parabola $y = -1/2x^2$

$x^2 = 4py$

$(y = -\frac{1}{2}x^2)-2$

$x^2 = -2y$

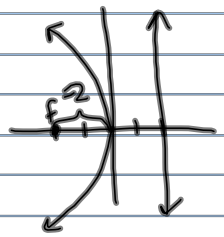


$4p = -2$

$p = -\frac{1}{2}$

$f(0, -1/2)$ directrix $y = \frac{1}{2}$ focal width $= 2$

Example 2: Find an equation in standard form for the parabola whose directrix is the line $x = 2$ and whose focus is the point $(-2,0)$



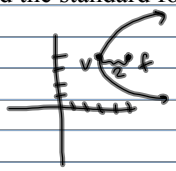
$y^2 = 4(-2)x$

$y^2 = -8x$

Translated Parabolas

$(x \pm h)^2 = 4p(y \pm k)$ or $(y \pm k)^2 = 4p(x \pm h)$

Find the standard form of the equation for the parabola with vertex (3,4) and focus (5,4)



$(y-4)^2 = 4(2)(x-3)$

$(y-4)^2 = 8(x-3)$

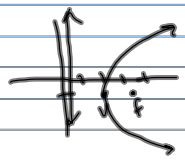
Example 5: given $y^2 - 6x + 2y + 13 = 0$ find its vertex, focus, and directrix

$y^2 + 2y + \_\_ = 6x - 13 + \_\_$

$(y+1)^2 = 6x - 12$

$(y+1)^2 = 6(x-2)$

$v(2,-1)$ focus $4p = 6$, $p = \frac{3}{2}$



directrix $x = \frac{1}{2}$

$(3\frac{1}{2}, -1) \Rightarrow (\frac{7}{2}, -1)$

Section 8.1 – Conic Sections & Parabolas

Assign: Read section 8.1 and do page 639 #1 – 10, 12 – 36 (*3) (Not 27 or 30), 49, 50, 54 – 56

Ex 11 - 30, find an equation in standard form for the parabola that satisfies the given conditions.

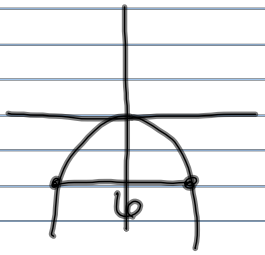
11. Vertex (0,0), focus (-3,0)

$y^2 = 4px$

$y^2 = 4(-3)x$

$y^2 = -12x$

19. Vertex (0,0), opens downward, focal width = 6

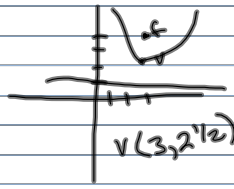


$6 = |4p|$

$6 = 4p$

$x^2 = -6y$

23. Focus (3,4), directrix y = 1



$\frac{4-1}{2} = \frac{3}{2} = p$

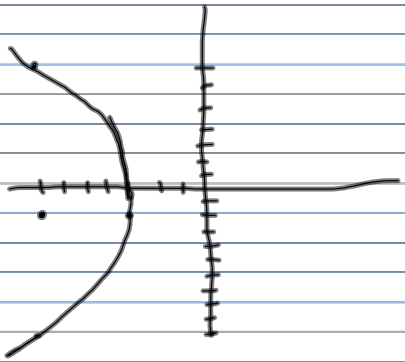
$V(3, 2\frac{1}{2})$ $(x-3)^2 = 4(3/2)(y - 5/2)$

$(x-3)^2 = 6(y - 5/2)$

In Exercises 31 - 36, sketch the graph of the parabola by hand.

34. $(y + 2)^2 = -16(x + 3)$

$V(-3, -2)$



In exercise 49 - 52, prove that the graph of the equation is a parabola, and find its vertex, focus, and directrix.

51. $y^2 - 4y - 8x + 20 = 0$

$y^2 - 4y + \underline{4} = 8x - 20 + \underline{4}$

$(y-2)^2 = 8x - 16$

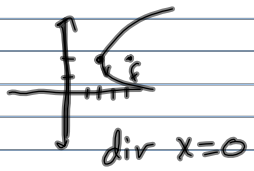
$(y-2)^2 = 8(x-2)$

v (2,2) focus

$4p = 8$

$p = 2$

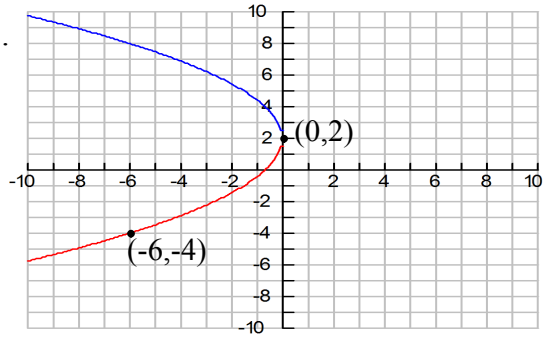
f (4,2)



dir $x = 0$

In exercises 53 - 56, write an equation for the parabola.

53.



$(y-2)^2 = 4px$

$(-4-2)^2 = 4p(-16)$

$(-6)^2 = 4p(-16)$

$36 = 4p(-6)$

$4p = -6$

$(y-2)^2 = -6x$