A Bibliography of Sign-Colored Graphs and Colored Matroids and Their Invariants

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A Mathematical Bibliography of Signed and Gain Graphs and Allied Areas
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Exp Expository.
Fr Frustration index (line index of imbalance) and other measures of imbalance.
Gen Generalization.
P All-negative or antibalanced signed graphs; parity-biased graphs.
Rand Random signs or gains, signed or gain graphs.
SG Signed graphs: mathematical properties.
T Topology applied to graphs; surface embeddings. (Not applications to topology.)

J. Akiyama, D. Avis, V. Chvátal, and H. Era

††1981a Balancing signed graphs. *Discrete Appl. Math.* 3 (1981), 227–233. MR 83k:05059. Zbl. 468.05066.

Bounds for $D(\Gamma)$, the largest frustration index $l(\Gamma,\sigma)$ over all signings of a fixed graph Γ (not necessarily simple) of order n and size $m = |E|$. Main Thm.: $\frac{1}{2}m - \sqrt{mn} \leq D(\Gamma) \leq \frac{1}{2}m$. Thm. 4: $D(K_{t,t}) \leq \frac{1}{2}t^2 - c_0 t^{3/2}$, where c_0 can be taken $= \pi/480$. Probabilistic methods are used. Thus, Thm. 2: Given Γ, $\operatorname{Prob}(l(\Gamma,\sigma) > \frac{1}{2}m - \sqrt{mn}) \geq 1 - (\frac{2}{e})^n$. Moreover, let $n_{\mathrm{b}}(\Sigma)$ be the largest order of a balanced subgraph of Σ. Thm. 5: $\operatorname{Prob}(n_{\mathrm{b}} K_n, \sigma) \geq k) \leq \binom{n}{k}/2^{\binom{k}{2}}$. (The problem of evaluating $n - n_{\mathrm{b}}$ was raised by Harary; see (1959b).) Finally, Thm. 1: If Σ has vertex-disjoint balanced induced subgraphs with m' edges, then $l(\Sigma) \leq \frac{1}{2}(m - m')$. [See Poljak and Turzík (1982a), Solé and Zaslavsky (1994a) for more on $D(\Gamma)$; Brown and Spencer (1971a), Gordon and Witsenhausen (1972a) for $D(K_{t,t})$; Harary, Lindström, and Zetterström (1982a) for a result similar to Thm. 1.] (**SG: Fr, Rand**)

Bela Bollobás

1978a *Extremal Graph Theory.* L.M.S. Monographs, Vol. 11. Academic Press, London, 1978. MR 80a:05120. Zbl. 419.05031.

A rich source of problems: find interesting generalizations to signed graphs of questions involving even or odd polygons, or bipartite graphs or subgraphs. (**p: X**)

§6.6, Problem 47, is the theorem on all-negative vertex elimination number from Bollobás, Erdős, Simonovits, and Szemerédi (1978a). (**p: Fr**)

B. Bollobás, P. Erdös, M. Simonovits, and E. Szemerédi

1978a Extremal graphs without large forbidden subgraphs. In: B. Bollobás, ed., *Advances in Graph Theory* (Proc. Cambridge Combin. Conf., 1977), pp. 29–41. Ann. Discrete Math., Vol. 3. North-Holland, Amsterdam, 1978. MR 80a:05119. Zbl. 375.05034.

Thm. 9 asymptotically estimates upper bounds on frustration index and vertex elimination number for all-negative signed graphs with fixed negative girth. [Sharpened by Komlós (1997a).] (**p: Fr**)

Thomas A. Brown and Joel H. Spencer

1971a Minimization of ± 1 matrices under line shifts. *Colloq. Math.* 23 (1971), 165–171. MR 46 #7059. Zbl. 222.05016.

Asymptotic estimates of $l(K_{r,s})$, the maximum frustration index of signatures of $K_{r,s}$. Improved by Gordon and Witsenhausen (1972a). Also, exact values stated for $r \leq 4$ [extended by Solé and Zaslavsky (1994a)]. (**sg: Fr**)

P.C. Fishburn and N.J.A. Sloane

1989a The solution to Berlekamp's switching game. *Discrete Math.* 74 (1989), 263–290. MR 90e:90151. Zbl. 664.94024.

The maximum frustration index of a signed $K_{t,t}$, which equals the covering radius of the Gale–Berlekamp code, is evaluated for $t \leq 10$, thereby extending results of Brown and Spencer (1971a). See Table 1. (**sg: Fr**)

Y. Gordon and H.S. Witsenhausen

1972a On extensions of the Gale–Berlekamp switching problem and constants of l_p-spaces. *Israel J. Math.* 11 (1972), 216–229. MR 46 #3213. Zbl. 238.46009.

Asymptotic estimates of $l(K_{r,s})$, the maximum frustration index of signatures of $K_{r,s}$, improving the bounds of Brown and Spencer (1971a). (**sg: Fr**)

R.L. Graham and N.J.A. Sloane

1985a On the covering radius of codes. *IEEE Trans. Inform. Theory* IT-31 (1985), 385–401. MR 87c:94048. Zbl. 585.94012.

See Example b, p. 396 (the Gale–Berlekamp code). (**sg: Fr**)

J.W. Moon and L. Moser

1966a An extremal problem in matrix theory. *Mat. Vesnik* 3 (18) (1966), 209–211. MR 34 #7385. Zbl. (e: 146.01401). (**sg: Fr**)

Patrick Solé and Thomas Zaslavsky

1994a A coding approach to signed graphs. *SIAM J. Discrete Math.* 7 (1994), 544–553. MR 95k:94041. Zbl. 811.05034.

Among other things, improves some results in Akiyama, Avis, Chvátal, and Era (1981a). Thm. 1: For a loopless graph with c components, $D(\Gamma) \geq \frac{1}{2}m - \sqrt{\frac{1}{2}\ln 2}\sqrt{m(n-c)}$. Thm. 2: For a simple, bipartite graph, $D(\Gamma) \leq \frac{1}{2}(m - \sqrt{m})$. *Conjecture.* The best general asymptotic lower bound is $D(\Gamma) \geq \frac{1}{2}m - c_1\sqrt{mn} + o(\sqrt{mn})$ where c_1 is some constant between $\sqrt{\frac{1}{2}\ln 2}$ and $\frac{1}{2}\pi$. *Question.* What is c_1 for, e.g., k-connected graphs? Thm. 4 gives girth-based upper bounds on $D(\Gamma)$. §5, "Embedded graphs", has bounds for several examples obtained by surface duality. All proofs are via covering radius of the cutset code of Γ. (**SG: Fr, T**)