Signed Graph Embeddings: Bibliography

Selections from

A Mathematical Bibliography of
Signed and Gain Graphs and Allied Areas

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Subject Classification Codes

A code in *lower case* means the topic appears implicitly but not explicitly. A suffix **w** on **SG**, **SD**, **VS** denotes signs used as weights, i.e., treated as the numbers $+1$ and -1, added, and (usually) the sum compared to 0. In a string of codes a colon indicates subcategorization.

A Adjacency matrix, eigenvalues.
Alg Algorithms.
Aut Automorphisms, symmetries, group actions.
B Balance (mathematical), cobalance.
Col Vertex coloring.
ECol Edge coloring.
Cov Covering graphs, double coverings.
D Duality (matroids or matrices).
E Enumeration of types of signed graphs, etc.
Exr Interesting exercises (in an expository work).
Exp Expository.
Fr Frustration index (line index of imbalance) and other measures of imbalance.
G Connections with geometry, including toric varieties, complex complement, etc.
Gen Generalization.
I Incidence matrix, Kirchhoff matrix.
K Signed complete graphs.
LG Line graphs.
M Matroids, chain-groups, flows.
N Numerical and algebraic invariants of signed graphs, etc.
O Orientations, bidirected graphs.
P All-negative or antibalanced signed graphs; parity-biased graphs.
Rand Random signs or gains, signed or gain graphs.
Ref Many references.
S Signed objects other than graphs and hypergraphs: mathematical properties.
SD Signed digraphs: mathematical properties.
SG Signed graphs: mathematical properties.
SH Signed hypergraphs: mathematical properties.
Str Structure theory.
Sw Switching of signs or gains.
T Topology of the graph, surface embeddings.
VS Vertex-signed graphs ("marked graphs"); signed vertices and edges.

Dan Archdeacon

1995a Problems in topological graph theory. Manuscript, 1995. WorldWideWeb URL (2/98) http://www.emba.uvm.edu/ãrchdeac/papers/papers.html

A compilation from various sources and contributors, updated every so often. "The genus sequence of a signed graph", p. 10: A conjecture due to Širáň (?) on the demigenus range (here called "spectrum" [though unrelated to matrices]) for orientation embedding of Σ, namely, that the answer to Question 1 under Širáň (1991b) is affirmative. **(SG: T)**

1996a Topological graph theory: a survey. Surveys in Graph Theory (Proc., San Francisco, 1995). *Congressus Numer.* 115 (1996), 5–54. Updated version: WorldWideWeb URL (2/98) http://www.emba.uvm.edu/ãrchdeac/papers/papers.html

Sect. 2.5 describes orientation embedding (called "signed embedding" [although there are other kinds of signed embedding]) and switching (called "sequence of local switches of sense") of signed graphs with rotation systems. Sect. 5.5, "Signed embeddings", briefly mentions Širáň (1991b), Širáň and Škoviera (1991a), and Zaslavsky (1993a, 1996a). **(SG: T: Exp)**

C. Paul Bonnington and Charles H.C. Little

1995a *The Foundations of Topological Graph Theory.* Springer, New York, 1995. MR 97e:05090. Zbl. 950.48477.

Signed-graph imbedding: see Sect. 2.3, Sect. 2.6 (esp. Thm. 2.4), pp. 44–48 (for the colorful 3-gem approach to crosscaps), Sect. 3.3, and Ch. 4 (esp. Thms. 4.5, 4.6). **(sg: T, b)**

André Bouchet

1982a Constructions of covering triangulations with folds. *J. Graph Theory* 6 (1982), 57–74. MR 83b:05057. Zbl. 488.05032. **(sg: O, T)**

A.M.H. Gerards, L. Lovász, A. Schrijver, P.D. Seymour, and K. Truemper

†1990a Manuscript in preparation, 1990.

See Seymour (1995a) for a very partial summary. **(SG: Str, M, T)**

Jonathan L. Gross and Thomas W. Tucker

1987a *Topological Graph Theory.* Wiley, New York, 1987. MR 88h:05034. Zbl. 621.05013.

Ch. 2: "Voltage graphs and covering spaces." Ch. 4: "Imbedded voltage graphs and current graphs." **(GG: T, Cov)**

Sect. 3.2.2: "Orientability." Sect. 3.2.3: "Rotation systems." Sect. 4.4.5: "Nonorientable current graphs", discusses how to deduce, from the signs on a current graph, the signs of the "derived" graph of the dual voltage graph. [The same rule gives the signs on the surface dual of any orientation-embedded signed graph.] (The sign group here is $\mathbb{Z}_2$.) **(SG: T)**

Elizabeth Klipsch

19xxa Some signed graphs that are forbidden link minors for orientation embedding. In preparation.

For each $n \geq 5$, either $-K_n$ or its 1-edge deletion, but not both, is a forbidden link minor. Which one it is, is controlled by Euler's polyhedral formula, provided $n \geq 7$. [A long version with excruciating detail is available.] **(SG: T, P)**

Sóstenes Lins

1981a A minimax theorem on circuits in projective graphs. *J. Combin. Theory Ser. B* 30 (1981), 253–262. MR 82j:05074. Zbl. 457.05057.

For Eulerian Σ in projective plane, max. number of edge-disjoint negative polygons = min. number of edges cut by a noncontractible closed curve that avoids the vertices. [Generalized by Schrijver (1989a).] **(SG: T, fr, Alg)**

1982a Graph-encoded maps. *J. Combin. Theory Ser. B* 32 (1982), 171–181. MR 83e:05049. Zbl. 465.05031, (478.05040).

See Sect. 4. **(sg: T: b)**

1985a Combinatorics of orientation reversing polygons. *Aequationes Math.* 29 (1985), 123–131. MR 87c:05051. Zbl. 592.05019. **(sg, p: T, Fr)**

Bojan Mohar

1989a An obstruction to embedding graphs in surfaces. *Discrete Math.* 78 (1989), 135–142. MR 90h:05046. Zbl. 686.05019.

The "overlap matrix" of a signed graph with respect to a rotation system and a spanning tree provides a lower bound on the demigenus that sometimes improves on that from Euler's formula. **(SG: T)**

Steven R. Pagano

1998a Separability and Representability of Bias Matroids of Signed Graphs. Ph.D. thesis, Dept. of Mathematical Sciences, Binghamton University, 1998.

Ch. 1: "Separability". Graphical characterization of bias-matroid k-separations of a biased graph. Also, some results on the possibility of k-separations in which one or both sides are connected subgraphs. **(GG: M: Str)**

Ch. 2: "Representability". The bias matroid of every signed graph is representable over all fields with characteristic $\neq 2$. For which signed graphs is it representable in characteristic 2 (and therefore representable over GF(4), by the theorem of Geoff Whittle, A characterization of the matroids representable over GF(3) and the rationals. *J. Combin. Theory Ser. B* 65 (1995), 222–261. MR 96m:05046. Zbl. 835.05015.)? Solved (for 3-connected signed graphs having vertex-disjoint negative polygons and hence nonregular matroid). There are two essentially different types: (i) two balanced graphs joined by three independent unbalanced digons; (ii) a cylindrical signed graph, possibly with balanced graphs adjoined by 3-sums. [See notes on Seymour (1995a) for definition of (ii) and for Lovász's structure theorem in the case without vertex-disjoint negative polygons.] **(SG: M: Str)**

Ch. 3: "Miscellaneous results". **(SG: M: Str, T)**

Gerhard Ringel

See also N. Hartsfield and M. Jungerman.

1977a The combinatorial map color theorem. *J. Graph Theory* 1 (1977), 141–155. MR 56 #2860. Zbl. 386.05030.

Signed rotation systems for graphs. Thm. 12: Signed rotation systems describe all cellular embeddings of a graph; an embedding is orientable iff its signature is balanced. Compare Stahl (1978a). Dictionary: "Triple" means graph with signed rotation system. "Orientable" triple means balanced signature. "Oriented" means all positive. **(SG: T, Sw)**

P.D. Seymour

See also Gerards *et al.* (1990a).

†1995a Matroid minors. In: R.L. Graham, M. Grötschel, and L. Lovász, eds., *Handbook of Combinatorics*, Vol. I, Ch. 10, pp. 527–550. North-Holland (Elsevier), Amsterdam, and MIT Press, Cambridge, Mass., 1995. MR 97a:05055. Zbl. 960.24825.

In Thm. 6.6, p. 546, interpreting G as a signed graph and an "odd-K_4" as a subdivision of $-K_4$ gives the signed graph generalization, due to Gerards and Schrijver (1986a). Let Σ be a signed simple, 3-connected graph in which no 3-separation has > 4 edges on both sides. Then Σ has no $-K_4$ minor iff either (i) deleting some vertex makes it balanced (the complete lift matroid of this type is graphic); or (ii) it is cylindrical: it can be drawn on a cylindrical surface that has a lengthwise red line so that an edge is negative iff it crosses the red line an odd number of times [Note: the extended lift matroid of this type is cographic, as observed by, I think, Gerards and Schrijver or by Lovász]. [See Pagano (1998a) for another use of cylindrical signed graphs.] [*Problem.* Find the forbidden topological subgraphs, link minors, and $Y\Delta$ graphs for cylindrical signed graphs.] [*Question.* Embed a signed graph in the plane with k distinguished faces so that a polygon's sign is the parity of the number of distinguished faces it surrounds. Cylindrical embedding is $k = 1$. For each k, which signed graphs are so embeddable?] **(SG: Str, T)**

Thm. 6.7 , pp. 546–547 generalizes to signed graphs, interpreting G as a signed graph and an "odd cycle" as a negative polygon. Take a signed simple, 3-connected, internally 4-connected graph. It has no two vertex-disjoint negative polygons iff it is one of four types: (i) deleting some vertex makes it balanced; (ii) deleting the edges of an unbalanced triangle makes it balanced; (iii) it has order ≤ 5; (iv) it can be orientation-embedded in the projective plane. This is due to Lovász; see, if you can, Gerards *et al.* (1990a). [A 2-connected Σ has no vertex-disjoint negative polygons iff $G(\Sigma)$ is binary iff $G(\Sigma)$ is regular iff the lift matroid $L(\Sigma)$ is regular. See Pagano (1998a) for classification of Σ with vertex-disjoint negative polygons according to representability of the bias matroid.] **(SG: Str, m, T)**

Jozef Širáň

1991a Characterization of signed graphs which are cellularly embeddable in no more than one surface. *Discrete Math.* 94 (1991), 39–44. MR 92i:05086. Zbl. 742.05035.

A signed graph orientation-embeds in only one surface iff any two polygons are vertex disjoint. **(SG: T)**

1991b Duke's theorem does not extend to signed graph embeddings. *Discrete Math.* 94 (1991), 233–238. MR 92j:05065. Zbl. 742.05036.

Richard A. Duke (The genus, regional number, and Betti number of a graph. *Canad. J. Math.* 18 (1966), 817–822. MR 33 #4917.) proved that the (orientable) genus range of a graph forms a contiguous set of integers. Stahl (1978a) proved the analog for nonorientable embeddings. Širáň shows this need not be the case for the demigenus range of an unbalanced signed graph. However, any gaps consist of a single integer each. The main examples with gaps are vertex amalgamations of balanced and uniquely embeddable unbalanced signed graphs, but a 3-connected example is $+W_6$ together with the negative diameters of the rim. *Question* 1 (Širáň). Do all gaps occur at the bottom of the demigenus range? [*Question* 2. Can one in some way derive almost all signed graphs with gaps from balanced ones?] **(SG: T)**

Jozef Širáň and Martin Škoviera

†1991a Characterization of the maximum genus of a signed graph. *J. Combin. Theory Ser. B* 52 (1991), 124–146. MR 92b:05033. Zbl. 742.05037.

The maximum demigenus $d_M(\Sigma)$ = the largest demigenus of a closed surface in which Σ orientation embeds. Two formulas are proved for $d_M(\Sigma)$: one a minimum and the other a maximum of readily computable numbers. Thus $d_M(\Sigma)$ has a "good" (polynomial) characterization. Along the way, several results are proved about single-face embeddings. *Problem* (Sect. 11). Characterize those edge-2-connected Σ such that Σ and all $\Sigma \backslash e$ have single-face embeddings. [A complex and lovely paper.] **(SG: T)**

Martin Škoviera

1992a Random signed graphs with an application to topological graph theory. In: Alan Frieze and Tomasz Luczak, eds., *Random Graphs, Vol. 2* (Proc., Poznań, 1989), Ch. 17, pp. 237–246. Wiley, New York, 1992. MR 93g:05126. Zbl. 817.05059.

The model: each edge is selected with probability p, positive with probability s. Under mild hypotheses on p and s, Σ is almost surely unbalanced and almost surely has a 1-face orientation embedding. **(SG: Rand, E, T)**

Saul Stahl

1978a Generalized embedding schemes. *J. Graph Theory* 2 (1978), 41–52. MR 58 #5318. Zbl. 396.05013.

A generalized embedding scheme for a graph is identical to a rotation system for a signing of the graph. Thm. 2: Signed rotation systems describe all cellular embeddings of a graph. Thm. 4: Embeddings are homeomorphic iff their signed rotation systems are switching equivalent. Thm. 5: An embedding is orientable iff its signature is balanced. Compare Ringel (1977a). Dictionary: λ is the signature. "λ-trivial" means balanced. **(sg: T, Sw)**

1978b The embeddings of a graph—a survey. *J. Graph Theory* 2 (1978), 275–298. MR 80a:05085. Zbl. 406.05027. **(sg: T)**

Andrew Vince

1983a Combinatorial maps. *J. Combin. Theory Ser. B* 34 (1983), 1–21. MR 84i:05048. Zbl. 505.05054.

See Theorem 6.1. **(sg: b: T)**

J.W.T. Youngs

1968a Remarks on the Heawood conjecture (nonorientable case). *Bull. Amer. Math. Soc.* 74 (1968), 347–353. MR 36 #3675. Zbl. 161.43303.

Introducing "cascades": current graphs with bidirected edges. A "cascade" is a bidirected graph, not all positive, that is provided with both a rotation system (hence it is orientation embedded in a surface) and a current (which is a special kind of bidirected flow). Dictionary: "broken" means a negative edge. [I omit work with current graphs because the results don't concern embedding of signed graphs; this is included just to show the connection with signed graph embedding.] **(sg: O: T, Flows)**

Thomas Zaslavsky

See also C. Greene, P. Hanlon, and P. Solé.

1981c Is there a theory of signed graph embedding? In: *Report on the XVIth Denison-O.S.U. Math. Conf.* (Granville, Ohio, 1981), pp. 79–82. Dept. of Math., The Ohio State Univ., Columbus, Ohio, 1981. **(()SG: T, M)**

1988b The demigenus of a signed graph. In: *Report on the XXth Ohio State-Denison Mathematics Conference* (Granville, Ohio, 1988). Dept. of Math., Ohio State Univ., Columbus, Ohio, 1988. **(()SG: T, M)**

1992a Orientation embedding of signed graphs. *J. Graph Theory* 16 (1992), 399–422. MR 93i:05056. Zbl. 778.05033.

Positive polygons preserve orientation, negative ones reverse it. The minimal embedding surface of a one-point amalgamation of signed graphs. The formula is almost additive. **(SG: T)**

1993a The projective-planar signed graphs. *Discrete Math.* 113 (1993), 223–247. MR 94d:05047. Zbl. 779.05018.

Characterized by six forbidden minors or eight forbidden topological subgraphs, all small. A close analog of Kuratowski's theorem; the proof even has much of the spirit of the Dirac-Schuster proof of the latter, and all but one of the forbidden graphs are simply derived from the Kuratowski graphs. [Paul Seymour showed me an alternative proof from Kuratowski's theorem that explains this; but it uses sophisticated results, as yet unpublished, of Robertson, Seymour, and Shih.] **(SG: T)**

[Related: "projective outer-planarity" (POP): embeddable in the projective plane with all vertices on a common face. I have found most of the 40 or so forbidden topological subgraphs for POP of signed graphs (finding the rest will be routine); the proof is long and tedious and will probably not be published. *Problem.* Find a reasonable proof.] **(SG: T)**

1995a The signed chromatic number of the projective plane and Klein bottle and antipodal graph coloring. *J. Combin. Theory Ser. B* 63 (1995), 136–145. MR 95j:05099. Zbl. 822.05028.

Introducing the signed Heawood problem: what is the largest signed, or zero-free signed, chromatic number of any signed graph that orientation embeds in the sphere with h crosscaps? Solved for $h = 1, 2$. **(SG: T, Col)**

1996a The order upper bound on parity embedding of a graph. *J. Combin. Theory Ser. B* 68 (1996), 149–160. MR 98f:05055. Zbl. 856.05030.

The smallest surface that holds K_n with loops, if odd polygons reverse orientation, even ones preserve it (this is parity embedding). That is, the demigenus $d(-K_n^\circ)$. **(P: T)**

1997a Is there a matroid theory of signed graph embedding? *Ars Combinatoria* 45 (1997), 129–141. MR 97m:05084. **(SG: M, T)**

1997b The largest parity demigenus of a simple graph. *J. Combin. Theory Ser. B* 70 (1997), 325–345. Zbl. 970.37744.

Like (1996a), but without loops. *Conjecture* 1. The minimal surface for parity embedding K_n is sufficient for orientation embedding of any signed K_n. *Conjectures* 3–4. The minimal surfaces of $\pm K_n^\circ$ and $\pm K_n$ are the smallest permitted by the lower bound obtained from Euler's polyhedral formula. **(P: T)**

1998a Signed analogs of bipartite graphs. *Discrete Math.* 179 (1998), 205–216.

Basically, they are the antibalanced and bipartite signed graphs; but the exact description depends on the characterization one chooses for biparticity: whether it is evenness of polygons, closed walks, face boundaries in surface embeddings, etc. Characterization by chromatic number leads to a slightly more different list of analogs. (**SG: Str, T**)

19xxa The largest demigenus of a bipartite signed graph. Submitted.
The smallest surface for orientation embedding of $\pm K_{r,s}$. (**SG: T**)