Problems in Signed, Gain, and Biased Graphs

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This is a fairly miscellaneous and incomplete selection of problems that I happen to have taken an interest in—not necessarily an active interest. Some are open and some are solved or partially solved—as for example a problem may have been shown to be NP-complete but special cases could still be solved exactly or algorithmically.

This list is intended to supplement the many problems in the Bibliography. There is just a small amount of duplication.

For the present, the problems here all concern signed graphs. However, many of them have obvious generalizations.

References are as cited in the Bibliography. All the terms employed should be defined in the Glossary. If you find any missing, or if you have suggestions for this page, please notify me!

Notation

- Γ denotes a graph, Σ a signed graph, Φ a gain graph with gain group $\mathfrak{G}$, and Ω a biased graph.
- $A(\Sigma)$ is the adjacency matrix.
- $G(\ )$ is the bias matroid, $L(\ )$ and $L_0(\ )$ are the lift and extended (or complete) lift matroids, of signed, gain, and biased graphs. $G(\Gamma)$ is the polygon matroid of a graph.

Basic References

[Bibliography] Thomas Zaslavsky, A mathematical bibliography of signed and gain graphs and allied areas, *Electronic J. Combin.* (1998), Dynamic Surveys in Combinatorics, No. DS8 (electronic). [MR 2000m:05001a. Zbl. 898.05001.]

[Glossary] ——, Glossary of signed and gain graphs and allied areas, *Electronic J. Combin.* (1998), Dynamic Surveys in Combinatorics, No. DS9 (electronic). [MR 2000m:05001b. Zbl. 898.05002.]

Contents

I. Direct Measures of Imbalance

[Date: June 8–10 1998.]

Imbalance of a signed graph can be measured in numerous ways. Here are problems concerning some measures that have appeared in the literature. The greatest interest has been in the edge version of frustration. (The problems in part II can be regarded as measuring imbalance in a different way.)

For the present, the problems here all concern signed graphs. However, many of them have obvious generalizations.

A. **Frustration index.**

Symbol: $l(\ )$. This is the least number of edges whose deletion results in a balanced subgraph. Its complement is the largest size of a balanced subgraph. It has turned out to be the measure that is most interesting and arises most often.

1. **Calculating $l(\Sigma)$.**
   Taking the all-negative signing, $l(-\Gamma)$ = the smallest size of the complement of a cut (bipartite subgraph). Thus the problem of calculating $l(\Sigma)$—more precisely, the question "$l(\Sigma) \leq k$?"—is at least as hard as that of finding a maximum-sized cut, which is a standard NP-complete problem. Barahona (1981a, 1982a) proved that the question is solvable in polynomial time if $|\Sigma|$ can be embedded in the torus.
   a. Is the question polynomial-time solvable if $|\Sigma|$ ranges over graphs embeddable in a fixed genus?
   b. Is it polynomial-time solvable if, while Σ ranges over all signed graphs, we are given an oracle that tells us:
      i. the genus of $|\Sigma|$?
      ii. an embedding of Γ in its minimal orientable surface?
2. **Most frustrated signatures.**
   Let $l(\Gamma) = \max l(\Gamma, \sigma)$ over all signings of a graph Γ.
   a. **Maximizing signatures.**
      Petersdorf's (1966a) Theorem says that $-K_n$ is the unique maximally frustrated signing of K_n.
      i. Which other graphs attain maximum frustration index when signed all negative?
      ii. Among these, for which is the all-negative signing uniquely most frustrated (up to switching, of course)?
      iii. In particular, is it true that chordal graphs are among the answers to the first two questions? This is plausible since they have lots of triangles.
      iv. Is it possible for a graph with even girth to attain maximum frustration when all negative? (Maybe the real question is whether girth has anything at all to do with whether the all-negative signature maximizes frustration.)
      v. What other kinds of graphs have nice answers to the question of which signings maximize the frustration index?
   b. **Estimates and values.**
      Akiyama, Avis, et al. (1981a) and Solé and Zaslavsky (1994a) found bounds on $l(\Gamma)$ for various kinds of graphs. Brown and Spencer (1971a), followed by Gordon and Witsenhausen (1972a), found asymptotic estimates for complete bipartite graphs. Brown and Spencer (1971a) found some exact values; also see Solé and Zaslavsky (1994a).

i. Improve the known bounds, both in general and for special types of graphs.
ii. Improve the asymptotic estimates for complete bipartite graphs.
iii. Find asymptotic estimates for other graphs.
iv. Evaluate $l(K_{r,s})$ for $r = 6, 7, \ldots$, extending the work of Brown and Spencer ($r < 5$) and Solé and Zaslavsky ($r = 5$). (Possibly a linear programming approach would be feasible for small r.)

3. **Restricted frustration index.**
Here we restrict the deleted edges to lie within a prescribed subset A. Let $l_A(\Sigma)$ be the smallest size of a subset D of A for which $l(\Sigma \setminus D) = l(\Sigma \setminus A)$ (the smallest frustration index one can hope to achieve). If $A = E$ this is $l(\Sigma)$. If $A = E_-$, the negative edge set, it is the *negative-edge frustration index* (proposed recently by Harary; the restricted frustration index is a generalization); here $l(\Sigma \setminus A) = 0$. Note that, unlike the frustration index, these numbers are not switching invariant. They are also NP-complete, at least when $A = E_-$ (from Σ form Σ' by the negative-subdivision trick; then $l(\Sigma') = l_{E_-}(\Sigma)$).
   a. For which subsets A does $l_A(\Sigma) = l(\Sigma)$? (Obviously it is never $< l(\Sigma)$.)
   b. In particular, find the signed graphs such that $l_{E_-}(\Sigma) = l(\Sigma)$.
   c. Find a use for $l_A(\Sigma)$. (It *looks* as though it ought to be good for something!)

B. **Vertex frustration number (vertex elimination number).**
Symbol: $l_0(\ )$. This is the smallest number of vertices whose deletion results in a balanced subgraph. Its complement is the largest order of a balanced subgraph. Taking the all-negative case, it becomes the problem of the largest order of a bipartite induced subgraph (whose NP-completeness I don't happen to know).

1. Can either of $l_0(\ )$ and $l(\ )$ be expressed in terms of the other?
2. Find any theorems about $l_0(\ )$.
3. Find any theorems about $l_0(\Gamma) = \max l_0(\Gamma, \sigma)$ over all signatures σ.

C. **Cycle indices of imbalance.**
In the early days of signed graph theory there was interest in the proportions of unbalanced polygons, balanced triangles, and balanced polygons weighted according to length (the weights decreasing with length). These were very hard to work with and not particularly natural as measures of imbalance. All I know of in successful mathematical treatments is some work of Norman and Roberts (1972a, b) on weighted measures.

1. Find mathematical properties of the proportion of unbalanced polygons, unbalanced triangles, or unbalanced polygons weighted by length.
2. Examine the proportion of unbalanced triangles for chordal graphs.
   a. Is it maximized by the all-negative signature (as is the case with complete graphs)?
   b. Are the maximizing signatures unique (up to switching)?

D. **Balancing degree.**
This is Δ_b = minimum of $\Delta(S)$ over all balancing edge sets S (that is, $\Sigma \setminus S$ is balanced). Hoffman's (19???) Theorem: There is a function f such that $\Delta_b(\Sigma) \leq f$(least eigenvalue of $A(\Sigma)$) when Σ is a signed complete graph.

1. Does this generalize to other (simple) graphs than complete graphs? To all simple graphs?
2. Find out something about Δ_b besides Hoffman's Theorem.

E. **Significant Subgraph Degrees.** (New)

Let $\bar{\Delta}_{\rm b} :=$ minimum of $\Delta(S)$ over all balancing edge sets S (that is, $\Sigma \setminus S$ is balanced). Note that $\bar{\Delta}_{\rm b}(\Sigma) \leq \frac{1}{2}\Delta(|\Sigma|$ is trivial.

Let $\Delta_{\rm b} :=$ minimum of $\Delta(S)$ over all balanced edge sets S (that is, $\Sigma\big|S$ is balanced).

1. **Eigenvalues.** Hoffman's Theorem: There is a function f such that $\bar{\Delta}_{\rm b}(\Sigma) \leq f(\lambda_1)$, where λ_1 is the least eigenvalue of $A(\Sigma)$, when Σ is a signed complete graph.
   a. Does this generalize to other (simple) graphs than complete graphs?
   b. To all simple graphs?
   c. Look for similar results about $\Delta_{\rm b}$.
2. **Extremal degrees.** (New 140998)
   a. Find interesting bounds on $\bar{\Delta}_{\rm b}$.
   b. Ditto for $\Delta_{\rm b}$.

II. Packing and Covering

[Date: June 8–10 1998; slightly revised July 2002.]

A. **Polygons.**

1. **Vertex disjoint.**
   a. Find the largest number of vertex-disjoint unbalanced polygons in a signed or biased graph (the *polygonal vertex-packing number*). There is a deep theorem in here somewhere.
   b. Find relations between the polygonal vertex-packing number and the frustration number (vertex elimination number) l_0. (In the contrabalanced case there is the Erdős-Posa theorem (1965a) bounding the latter in terms of the former.)
   c. Characterize the signed, or biased, graphs for which this number is at most k. (The signed case $k = 1$ has been solved by Lovász (unpublished). The contrabalanced case $k = 1$ is Lovász's theorem on the nonexistence of vertex-disjoint polygons in a graph.)
   d. Obviously this number is not greater than $l_0(\Sigma)$. When is it equal? (This, in general or for signed graphs, would be a deep theorem. Solved for contrabalanced graphs by Ding.)
2. **Edge disjoint.**
   a. Find the largest number of edge-disjoint unbalanced polygons (the *polygonal edge-packing number*). There is also a deep theorem in here.
   b. Find relations between the polygonal edge-packing number and the frustration index l. (In the contrabalanced case there is the Erdős-Posa theorem bounding the latter in terms of the former.)
   c. Characterize the signed, or biased, graphs for which this number is at most k.
   d. Obviously this number is not greater than $l(\Sigma)$. When is it equal? (This, in general or for signed graphs, would be a deep theorem and very interesting. Solved for contrabalanced graphs by Ding.)

B. **Balanced Decomposition.**

A balanced decomposition is a decomposition of Σ into balanced subgraphs. It is *minimal* if it decomposes Σ into the fewest possible subgraphs. (Decomposition means that the edges are partitioned amongst the subgraphs. Every subgraph has all the vertices.) This generalizes bipartite decomposition of unsigned graphs (take all signs negative) and forest decomposition (take all polygons unbalanced). See Zaslavsky (1987b).

1. **Balanced decomposition number.**
   The balanced decomposition number is the smallest number of subgraphs in a balanced decomposition. There is a simple formula for signed graphs (Zaslavsky) and a famous formula for contrabalanced biased graphs (Nash-Williams). Find something for all biased graphs. Slilaty has shown that this is hard (unpublished).
2. **Connected balanced decomposition number.**
   The connected balanced decomposition number (c.b.d.n.) is similar but requires connected subgraphs.
   a. Find out something about the c.b.d.n. of a general signed graph.
   b. Find the c.b.d.n.'s of signings of particular graphs.
   c. Study the maximum c.b.d.n. of all signings of a graph.
   d. Study the c.b.d.n. of a biased graph.
3. **Balanced decomposition degree.** (New)
   For a balanced decomposition ρ of Σ into subgraphs S_i, let $\Delta_{\max}(\rho)$ be the maximum $\Delta(S_i)$ (where Δ denotes maximum degree of a graph) and let $\Delta_{\min}(\rho)$ be the minimum. Over all minimal balanced decompositions ρ of Σ, let $\Delta_{\max}(\Sigma)$ = the maximum of $\Delta_{\max}(\rho)$. These are measures of the degree to which minimal balanced decompositions divide the edges up equitably among the subgraphs.
   a. What can one say about $\Delta_{\max}(\Sigma)$?
   b. What about the minimum of $\Delta_{\max}(\rho)$ over all minimal balanced decompositions of Σ?
   c. What about the maximum and minimum of $\Delta_{\max}(\Sigma)$ over all signings of a particular graph?
   d. What can one say about $\Delta_{\mathrm{dmin}}(\Sigma)$? (New 140998.)
   e. It must be obvious that I'm not sure which are the best numbers to look at in order to get a good picture of how equitable a balanced decomposition can be. Figure out which one(s) make the most sense.

C. **Covering by balancing edge sets** (New)

1. Consider the intersection of all maximal balanced subsets. Equivalently (by complementation) this is the union of all balancing edge sets.
   a. When is it empty? (When does the union of all balancing edge sets cover Σ?)
   b. In general what is it? That is, which edges belong to every maximal balanced subgraph?
2. What is the smallest number of balancing edge sets required to cover the union of all balancing edge sets? Especially, when these sets cover Σ.

III. State Spaces for Signed Graphs

(New 12 September 1998 and preceding couple of weeks.) (Originated by Thomas Zaslavsky, inspired by (mis)reading the SK model and in particular the book of Mézard, Parisi, and Virasoro.)

A "state" of a signed graph is an assignment of a sign to each vertex; thus it is equivalent to a switching function. (There are generalizations in which each vertex is assigned something from a set on which the sign group acts—then we have a "permutation signed graph"—but I will only discuss signs here; the problems are difficult enough.) The *state space* of Σ is the set of all states, connected by the Hamming graph: i.e., states are adjacent when they differ at a single vertex. A *ground state* of Σ is a state s in which the fewest edges are frustrated (i.e., negative in Σ^s); equivalently, $h(s) = 0$, where

$$h(s) = |E_-(\Sigma^s)| - l(\Sigma)$$

is the *altitude* of s.

The two state spaces of primary concern are the state space and the ground state space, the subspace of all ground states.

These problems are suggested by my reading (and derandomizing) probabilistic studies of the Sherrington–Kirkpatrick Ising model in spin glass theory, although they probably do not have direct significance for that model. [The Hamming metric is the one employed by the physicists, although I don't understand why they use it instead of the path metric, which occurred to me as the obvious one for their purposes concerning energy valleys and barriers (and still does, which shows how well I understand them). It also isn't clear to me exactly what they mean by a "state": sometimes it's clearly a spin assignment, other times ("pure states") it seems to be something else.—13 Sept 1998]

A. The Hamming Metric.

The distance between states is given by either of two equivalent measures: the Hamming distance in state space,

$$d_H(s, s') = |(ss')^{-1}(-)|,$$

that is the number of vertices at which the states differ, or the normalized distance index

$$q(s, s') = |V|^{-1} \sum_v s_v s'_v$$

(known in spin glass theory as the "overlap" of two replicas). The problem is to study this metric on the ground state space of some signed graphs. Nothing is known. Exact calculations seem virtually impossible except in small examples—see Grötschel, Jünger, and Reinelt(1987a)—or special cases—e.g. complete graphs: Petersdorf (1966a).

1. **The ground state space.**
   a. **Theory.**
      i. Prove some properties of the Hamming metric.
   b. **Computations.**
      i. Calculate the ground state space of some nontrivial signed graph.
      ii. Compare the ground state metrics of all signings of a very small graph.
      iii. The same for a selection (random or otherwise) of signings of a larger graph.
   c. **Complete graphs.**
      i. Find the exact metric structure for all-negative complete graphs.
      ii. Compare with some other signatures (not switching equivalent to all negative).

2. **Low-altitude state spaces.**
   These are the spaces of all states that have altitude at most h_0, a constant.
   a. The same questions as above, about the low-altitude state spaces.
   b. Compare the low-altitude state spaces as the maximum altitude h_0 increases from 0.
      i. How fast does the space grow?
      ii. Is there something significant about which states are added as h_0 increases?

B. **The Edge Metric.**
The distance function is

$$d(s, s') = h(ss'),$$

the number of edges on which Σ^s and $\Sigma^{s'}$ differ. (This is the Hamming metric on signatures of the underlying graph, restricted to a single switching class.) The remarks about difficulty of computation in the Hamming metric apply equally.

1. **The state space.**
   a. **Theory.**
      i. The distance between s and $-s$ is 0; therefore this is really a metric on the state space modulo negation (on each component, for a disconnected graph). What are the implications? In particular, verify that we have a metric.
      ii. Prove some intrinsic properties of the edge metric.
      iii. Compare the edge metric to the Hamming metric. How does the answer depend on Σ?
   b. **Computations.**
      i. Calculate the metric of the state space of some nontrivial signed graph.
      ii. Compare the state spaces of all signings of very small graphs.
      iii. The same for a selection (random or otherwise) of signings of a larger graph.
   c. **Complete graphs.**
      i. Find the exact metric structure for all-negative complete graphs.
      ii. Compare with some other signatures (not switching equivalent to all negative).
2. **The ground state space.**
3. Solve the same problems as for the state space. These problems are technically included in those for the state space but larger graphs might be handled.

C. **The Path Metric.**
The distance function is

$$d(s, s') = \min_P \max_{s^*} h(s*) - \min\{h(s), h(s')\},$$

the minimum taken over all ss' paths P in the Hamming graph of states and the maximum over all states s^* on P.

The remarks about difficulty of computation in the Hamming metric apply more strongly than ever. Despite the difficuly of calculation, this is perhaps the most interesting metric since it embodies the height of energy barriers to getting from one ground or low-altitude state to another. For that purpose it might be more appropriate to measure height by the exponential altitude

$$h_{\exp}(s) = e^{h(s)}.$$

1. **The state space.**
   Solve the same problems as for the edge metric.
2. **The ground state space.**
   The same problems as for the state space. These problems are technically included in those for the state space but larger graphs might be feasible.

D. **Frustration in States**

(New 28 August 1998, 12 September 1998) (Originated by Thomas Zaslavsky, inspired

by the SK model and in particular the book of Mézard, Parisi, and Virasoro.)

1. **General Definition.**
   Define a state, the (weighted) frustration of a state, and its negative the Hamiltonian as generally as possible so as to handle vector sign and gauge models. Presumably one needs some sort of form (bilinear? whatever that might mean) on spin pairs, $\langle s_i, s_j\rangle$, which can then be applied to calculate the weighted frustration of the state as

$$\sum_{e_{ij}} \langle s_i\varphi(e_{ij}), s_j\rangle\ w(e_{ij}).$$

   For instance when the spin set is S^d the form is inner product, i.e., $\cos(\angle)$.
   Here we have for the frustration of an edge a measure that need not be 1 or 0 (as it is in a signed graph if we define the frustration of e to be $\frac{1}{2}(1-\sigma^s(e))$.
   I would like to generalize the latter statement about frustration of an edge in a signed graph. However, in the generalization with spins in S^d, the spin set and the gain group $\mathfrak{O}(n)$ are totally different sets.

IV. Geometry of Gain Graphs

A. **Pappus' Theorem.** (July 31, 2000.)

A theorem of Pappus states that, for a triangle $\triangle B_0B_1B_2$, inscribed in $\triangle A_0A_1A_2$ with B_i on edge A_jA_k, to have centroid equal to that of $\triangle A$, it suffices that the Cevian coordinates (see Zaslavsky (20xxf), §2) $\varphi^*(B_0; A_1, A_2)$, $\varphi^*(B_1; A_2, A_0)$, $\varphi^*(B_2; A_0, A_1)$ be all equal. (See [1, p. 302, note 2].) The condition seems necessary as well. Pappus' theorem clearly remains valid for B_i anywhere on the extended edge A_jA_k. (We admit ∞ as a possible value of Cevian coordinate.) One wonders:

1. Is there a nice proof using Cevian coordinates?
2. What multidimensional generalization there may be? Here is one suggestion. *Conjecture.* If $A_0, A_1, \dots, A_d$ are the vertices of a simplex s in $\mathbb{E}^d$ and if B_{01}, B_{12}, $\dots$, B_{d0} are chosen on the extended edges $A_0A_1, A_1A_2, \dots, A_dA_0$, respectively, then $s' = \operatorname{conv}(B_{01}, B_{12}, \dots, B_{d0})$ has the same centroid as does s if and only if all $\varphi^*(B_{i,i+1};$ $A_i, A_{i+1})$ are equal (subscripts modulo $d+1$).

Possibly the answers to my questions are already known. I have not yet tried to search the literature.

B. **Unification of Ceva and Clement.** (September 20, 2000.) This problem was suggested by Ethan Bolker. He has found that Walter Whiteley can convert any problem he mentions involving right angles into a projective property. He asks:

1. *Question.* Can Ceva's theorem and "Clement's Theorem" be unified in this way? We supposed that there would have to be an absolute conic (or whatever it's called) to make the right angles. Possibly "Clement" comes from Ceva with a special choice of the absolute?

V. Matroids

(December 6–9, 2002.)

Most of these problems come in "flavors" and "colors": they can be asked about either signed graphs or general biased graphs (or gain graphs with a particular gain group; I will not mention these separately as a rule, but especially $\mathbb{Z}_3$ should be born in mind), and they can be asked with reference to the bias or lift matroid. Signed graphs, or gain graphs with a specified small gain group, can be expected to give simpler answers, but usually there seems no reason to expect the same of gain graphs in general. One expects especially simple answers for full graphs. (Extended lift matroids are essentially the lift matroids of full graphs.)

A. **Characterization** (July 2002; December 6, 2002.)

1. Which matroids are bias matroids of signed graphs? Of full signed graphs?
2. Which matroids are bias matroids of biased graphs? Of full biased graphs?
3. Which matroids are lift matroids of signed graphs? Extended lift matroids?
4. Which matroids are lift matroids of biased graphs? Extended lift matroids?
5. The same for gain graphs with a specific gain group, especially $\mathbb{Z}_3$.

Slilaty and Abrams have some unpublished recent results on signed graphs.

B. **Isomorphism** (July 2002; December 6, 2002.)

There are three basic problems.

1. Decide whether two given signed, gain, or biased graphs have isomorphic bias matroids.
2. Decide whether they have isomorphic lift matroids.

C. **Connectivity** (July 2002; December 6, 2002.)

1. Compare the connectivity of the (signed or biased) graph to that of the bias or lift matroid.
2. Is there a reasonable notion of connectivity of a signed or biased graph that is different from the connectivity of the underlying graph but is not merely a restatement of matroid connectivity? (I have not found it necessary to develop such a notion.)

D. **Special types of matroids** (July 2002; December 6, 2002.)

The problem is to characterize signed, gain, or biased graphs whose matroid is in a certain class; in particular, by forbidden minors.

1. Which signed or biased graphs have bias matroids that are graphic? cographic? Dowling–Qin and Slilaty have some recent results on forbidden minors for cographicity of signed graphs and more generally of gain graphs with even-order gain groups.
2. Which signed or biased graphs have lift matroids that are graphic? cographic?

VI. Random Gain Graphs

(August 30, 2000.)

A. **Balanced and identable subgraphs**

Thomason (1988a) suggests a general question about asymptotic probability of a property of a bond or cut. In connection with cuts I would regard a graph as signed all negative; thus a cut is the positive subgraph in a switching of $-\Gamma$. Thus the generalization is to the identity-gain subgraph in some $\mathfrak{G}$-switching of Φ, a $\mathfrak{G}$-gain graph. I'll call this an *identable* subgraph of Φ. This includes all maximal balanced subgraphs but is not limited to them. A bond is a minimal cut so the analog is minimal identable subgraphs. Some problems about this appear here.

Given a property P of an ordinary, simple graph (no signs or gains), let $\pi_n^*(P;\mathfrak{G})$ = probability that a random balanced subgraph of $\mathfrak{G}K_n$ has property P. Let $\hat\pi_n(P;\mathfrak{G})$ = probability that a random $\mathfrak{G}$-gain graph Φ of order n (without loops) has an identable subgraph with property P. Let $\hat\pi'_n(P;\mathfrak{G})$ be the same but restricted to minimal identable subgraphs. Let $\hat\pi^*_n(P;\mathfrak{G})$ be the same but restricted to maximal balanced subgraphs (equivalently, maximal identable subgraphs).

1. **Independence of Group.** (August 30, 2000.)
   a. How does π_n^* vary as a function of $\mathfrak{G}$? (This is the real problem.) *Conjecture*: It is a function of $|\mathfrak{G}|$ alone, no matter what reasonable property P may be. (It may be necessary to give an eccentric definition of "reasonable property" in order to make this work.)
   b. The same for the asymptotic value $\pi^*(P;\mathfrak{G}) = \lim_n \pi_n^*(P;\mathfrak{G})$. Of course if my π_n^*-conjecture is correct, the asymptotic conjecture is correct.
   c. Do $\hat\pi_n, \hat\pi'_n, \hat\pi^*_n$ differ for any nontrivial properties? (This is a purely technical question that determines how many variants there are of the real questions.)
   d. How does $\hat\pi_n$ vary as a function of $\mathfrak{G}$? (This is the real problem.) *Conjecture*: It is a function of $|\mathfrak{G}|$ alone, no matter what reasonable property P may be. (It may be necessary to give an eccentric definition of "reasonable property" in order to make this work.)
   e. The same for the asymptotic values $\hat\pi(P;\mathfrak{G}) = \lim_n \hat\pi_n(P;\mathfrak{G})$, etc. Of course if my $\hat\pi_n$-conjecture is correct, the asymptotic conjecture is correct.

   Note that the probability of an edge should be constant since all edges are equivalent under switching automorphisms of the universal graph $\mathfrak{G}K_n$.
2. **Asymptotics of Particular Properties.** (August 30, 2000.) Evaluate the (asymptotic) probabilities for particular P.
   a. Thomason's property: Being Eulerian (in whatever variant definition one chooses). (August 30, 2000.)

References

[1] E.J. Dijksterhuis, *Archimedes.* Acta Hist. Sci. Nat. et Med., Edidit Bibliotheca Univ. Hauniensis, Vol. 12. Ejnar Munksgaard, Copenhagen, 1956. MR 18, 981g. Reprinted, with a contribution by Wilbur R. Knorr. Princeton University Press, Princeton, N.J., 1987. MR 92g:01011. Zbl. 646.01001.