# Corrigenda for OESG
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Sect. 7, pp. 406–407. The sign of $W$ is reversed. It should be: On p. 406, l. -2 and p. 407, l. 2: $\sigma(W) = -\alpha\beta$. On p. 407, l. 1: $\alpha = -\beta$ (not $\alpha = \beta$).

Sect. 7, p. 409. Par. 4, l. 4: $k$ should be $l$. Par. 5, l. 4: "green edges $\hat{z}_m\hat{x}_1$ and $\hat{y}_m\hat{w}_1$."; ll. 5–6: "green edges $\hat{y}_m v_n$, $u_{m+1}\hat{w}_1$, and $\hat{z}_m\hat{x}_1$. … we take $v_n u_1$."

Sect. 7, p. 410. Par. -3, l. 4: $D_j$ instead of $C_j$ (twice).

Sect. 7, p. 411. Ll. 1–2 in the text: $R \cup B$ instead of $R \cup G$ (twice). Par. -2, l. 1: Replace "In $\mathbf{R}'_i$," by "In the local graph of $\mathbf{R}'_i$ at $v$,".

Sect. 8, p. 413. There are several problems with the proof of Theorem 8.1, Step 2.

L. 1: insert the sentence: (By choosing a new $G_i$ we change $\mathbf{R}_i$ to $\mathbf{R}'_i$.)

In the first displayed equation:

$$\begin{aligned}
=&\{\deg_{\Sigma_1}(v) + \epsilon_1 - [\tfrac{1}{2}\#R_{1-} + c_1(B_1 \cup R_1)]\} \\
&+ \{\deg_{\Sigma_2}(v) + \epsilon_2 - [\tfrac{1}{2}\#R_{2-} + c_1(B_2 \cup R_2)]\} \\
=&\{\deg_{\Sigma}(v) + \epsilon_1 + \epsilon_2 - [\tfrac{1}{2}\#R_{-} + c_1(B \cup R)]\}
\end{aligned}$$

Par. 2, l. 2: $S_i = S(\mathbf{R}'_i)$.

Par. 3 (the last of the proof): There is a gap here. The problem is that $\epsilon_1$ and $\epsilon_2$ depend on the new subgraph embeddings, i.e., on the $\mathbf{R}'_i$. These may both be minimal, as assumed in the proof, but it is also possible that only one is minimal, the other is almost minimal, and $v$ is loopable in both embeddings. To obtain a valid proof, replace the paragraph, after the first sentence, by the following reasoning.

We know from Step 1 that $\delta \leq d(\Sigma_1) + d(\Sigma_2) - d(\Sigma)$. We also know that $\epsilon = \epsilon_1\epsilon_2$, because $R$ consists of the same edges with the same signs as $R_1 \cup R_2$. Therefore we have the inequalities

$$\delta \leq d(\Sigma_1) + d(\Sigma_2) - d(\Sigma) \leq d(S_1) + d(S_2) - d(S(\Sigma)) = (1 - \epsilon_1)(1 - \epsilon_2). \quad (8.A1)$$

In case both $\mathbf{R}'_1$ and $\mathbf{R}'_2$ are minimal embeddings, the inequalities in (8.A1) are actually equalities; for if $\epsilon_1 = \epsilon_2 = 0$, then $v$ is loopable in both embeddings so $\delta = 1$; while if either $\epsilon_1$ or $\epsilon_2 = 1$, then (8.A1) forces $\delta = 0$. Thus the theorem holds. On the other hand, if the $\mathbf{R}'_i$'s are not both minimal, then the second inequality in (8.A1) is strict. It follows that $\delta = 0$ and $\epsilon_1 = \epsilon_2 = 0$, whence

$$d(S_1) + d(S_2) - 1 = d(\Sigma_1) + d(\Sigma_2) = d(\Sigma).$$

Thus both subgraphs $\Sigma_i$ are embedded in their surfaces $S_i$ so that $v$ is loopable, and for one of the subgraphs the embedding is minimal; while for the other, we have $d(S_j) = d(S(\Sigma_j)) + 1$ and, since $\delta = 0$, $\Sigma_j$ has no minimal embedding in which $v$ is loopable. Therefore again the theorem is verified. This completes the proof.

*Example 8.A1.* The two cases at the conclusion of Step 2 really can occur. Here is an example in which a minimal embedding of $\Sigma$ gives rise to a minimal embedding of $\Sigma_1$ and a nonminimal embedding of $\Sigma_2$. We let $\Sigma_1 = -K_4$, switched so that a quadrilateral is positive, and $\Sigma_2 = K_4(vw)$, a signed $K_4$ with one negative edge $vw$. Minimal embeddings of $\Sigma_1$ and $\Sigma_2$ in $U_1$ are contained (joined at the vertex $v$) in Figure A2; clearly, in the latter no vertex is loopable. It is not hard to see that $\Sigma_2$ has a unique minimal embedding; it follows that $d(\Sigma) \geq 2$. An embedding of $\Sigma$ in $U_2$ is shown in Figure A1. Restricted to $\Sigma_1$ and with the unused crosscap deleted, this is a minimal embedding (in $S_1 = U_1$) in which $v$ is loopable; restricted to $\Sigma_2$, it is a nonminimal embedding (in $S_2 = U_2$) in which $v$ is loopable. As the proof predicts, we have $d(\Sigma) = d(S_1) + d(S_2) - 1$, the subtraction due to the loopability of $v$ in the subgraph embeddings.

A different minimal embedding of $\Sigma$ is obtained by combining minimal embeddings of $\Sigma_1$ and $\Sigma_2$. Figure A2 shows the result.

Note that we would have gotten a valid example by taking $\Sigma_2$ to be just a negative loop!

Sect. 8, p. 414. Par. -1, l. 1: "Suppose $v$ is loopable in $\Sigma_1$ and $\Sigma_2$ as embedded in $S_1$ and $S_2$."

Sect. 8, p. 415. Par. 1, display: The subscripts on the loops, $l'_1$ and $l'_2$, should be reversed. Par. 2, l. -2: The first red-green polygon should read $g_{21} P_1 g*_{21}^{-1} Q_2$.

Sect. 10, p. 420. In (10.2), the first inequality should be $\geq$, not $\leq$. (Note that equality holds here, since $\leq$ holds whenever a single edge is being added to a graph.) In (10.3), the first inequality should read

$$d(\Sigma_i \backslash e_i) \leq d(\Sigma_i) - 2.$$

Reference updates:

[20] *Ars Combinatoria* **45** (1997) 129–141.
[21] *Discrete Math.* **113** (1993) 223–247.

CAPTIONS

Figure A1. A minimal embedding of the signed graph in Example 8.A1 which is not a combination of minimal embeddings of its two blocks.

Figure A2. A minimal embedding of the same graph which is a combination of minimal embeddings of its two blocks. Each block can be seen to be embedded in $U_1$.